5/17/18 Lecture outline

⋆ Reading: Zwiebach chapter 9.

• Continue where we left off last time: recall ($\hbar = c = 1$ units, $[\alpha'] = 1/[T_0] = L^2$)

$$\mathcal{L}_{NG} = -\frac{1}{2\pi\alpha'}\sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2},$$

and we have

$$\mathcal{P}^\tau_\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} = -\frac{1}{2\pi\alpha'}\frac{(\dot{X}\cdot X')X'_\mu - (X')^2\dot{X}_\mu}{\sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2}},$$

and

$$\mathcal{P}^\sigma_\mu = \frac{\partial \mathcal{L}}{\partial X^{\mu\prime}} = -\frac{1}{2\pi\alpha'}\frac{(\dot{X}\cdot X')\dot{X}_\mu - (\dot{X})^2 X'_\mu}{\sqrt{(\dot{X}\cdot X')^2 - (\dot{X})^2(X')^2}}.$$

which we simplified by picking static gauge.

• Generalize static gauge (to eventually get to light cone gauge). Consider e.g. gauge $n_\mu X^\mu = \lambda\tau$ for time-like n_μ. Static gauge is $n_\mu = (1, 0, \ldots, 0)$. Vary, $n_\mu dX^\mu = \lambda d\tau$, so n_μ is orthogonal to the string tangent at constant τ. We want dX^μ along the string to be spacelike (or null at isolated points, e.g. the Neumann open string endpoints).

For open strings in natural units we can take $\lambda = 2\alpha'(n\cdot p)$.

Static gauge implies that $n_\mu \mathcal{P}^{\tau\mu}$ is a constant. The generalization for general n^μ is that $n\cdot\mathcal{P}^\tau$ is a constant of the motion of the string worldsheet. This is not a reprarameterization invariant statement - that is the point: we are using it to fix a gauge.

Using the EOM, this implies that $n\cdot\mathcal{P}^\sigma$ is independent of σ and then can argue that $n\cdot\mathcal{P}^\sigma = 0$.

More generally, it is convenient to write the gauge fixing conditions as

$$n\cdot\mathcal{P}^\sigma = 0, \qquad n\cdot X = \beta\alpha'(n\cdot p)\tau, \qquad n\cdot p = \frac{2\pi}{\beta} n\cdot\mathcal{P}^\tau,$$

where $\beta = 2$ for open strings and $\beta = 1$ for closed strings. These lead to

$$\dot{X}\cdot X' = 0 \qquad \dot{X}^2 + c^2 X'^2 = 0. \tag{1}$$

$$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\dot{X}^\mu \qquad \mathcal{P}^{\sigma\mu} = -\frac{c^2}{2\pi\alpha'}X^{\mu\prime}, \tag{2}$$

$$(\partial_\tau^2 - c^2\partial_\sigma^2)X^\mu = 0. \tag{3}$$

• We will later focus on light cone gauge: $n_\mu = (1/\sqrt{2}, 1/\sqrt{2}, 0, \ldots)$. Introducing n^μ obscures the relativistic invariance in spacetime. Why would we want to do that? Well we wouldn't, except that it happens to have some other benefits once we quantize the theory. It gives a way to determine the spectrum without having to introduce unphysical states. There is a covariant approach, but it requires introducing unphysical states ("ghosts") and then ensuring that they are projected out of the physical spectrum – doing this requires sophisticated theory which is only taught at the advanced graduate student level, so we'll stick with the simpler (and in the end physically equivalent) light-cone gauge description.

• The general solution of the linear equations (3) is a superposition of Fourier modes

$$X^\mu(\tau,\sigma) = x_0^\mu + 2\alpha' p^\mu \tau + i\sqrt{2\alpha'}\sum_{n\neq 0}^{\infty} \frac{1}{n}\alpha_n^\mu e^{-in\tau}\cos n\sigma,$$

where $\alpha_{-n}^\mu \equiv \alpha_n^{\mu *}$ (to make X^μ real) and it's also convenient to define $\alpha_0^\mu \equiv \sqrt{2\alpha'}p^\mu$. Then

$$\dot{X}^\mu \pm X^{\mu'} = \sqrt{2\alpha'}\sum_{n=-\infty}^{\infty} \alpha_n^\mu e^{-in(\tau\pm\sigma)}.$$