154 Homework 2, due 4/12/16

1. Thomson 17.4.
2. Verify that $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu$ is invariant under the gauge transformation. This is similar to exercise 17.3 in Thomson, but I'd like you to include the $j^\mu A_\mu$ term and verify that both that and the other term are separately gauge invariant; the gauge invariance of the $j^\mu A_\mu$ term uses the fact that $\partial_\mu j^\mu = 0$.
3. Thomson 17.7.
4. Thomson 3.1.
5. In class, I discussed how to use $\psi \sim e^{iS/\hbar}$ to show that a particle of electric charge q_{elec}, circling a solenoid of magnetic flux Φ, picks up a phase factor. Writing $\Phi = 4\pi q_{mag}$ (a magnetic analog of Gauss' law), complete the argument that I started, due to Dirac, that the Dirac string (pretend solenoid) is only unobservable if $q_{elec}q_{mag}$ takes some quantized values. Write out the quantization condition, including all factors of $\hbar$ and c explicitly (don't set them to 1 here).
6. Verify that $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu + \frac{1}{2}m_\gamma^2 A_\mu A^\mu$ violates gauge invariance if $m_\gamma \neq 0$. (Moral: gauge invariance forbids a photon mass. Caveat: there is a way to get around this with a bose condensate. This is what happens in the Higgs mechanism, for the Weak forces. And it is what happens in a superconductor for E and M.)