154 Homework 1, due 4/5/16

1. Use the chain rule to show that $\partial f/\partial x^\mu$ transforms as a 4-vector with lowered index. Evaluate $\partial_\mu(x_\nu x^\nu)$ and $\partial_\mu x^\mu$.

2. Verify that $\phi \equiv e^{\pm i k_\mu x^\mu}$ is a plane wave solution of $\partial_\mu \partial^\mu \phi = 0$ provided that k_μ satisfies a certain condition – and write out that condition. Suppose e.g. that $\vec{k} = (1, 2, 3)$; then what is k^0?

3. Verify that half of Maxwell's equations are solved by taking $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, and that the other half then take the form $\partial_\mu F^{\mu\nu} = J^\nu$. Verify that the Lorentz force law can be written as the space component of the 4-vector equation $dp^\mu/d\tau = (q/c)F^{\mu\nu}u_\nu$, where $u^\mu = dx^\mu/d\tau$.

4. Verify that charge conservation can be written as $\partial_\mu J^\mu = 0$, and that this is an automatic consequence / requirement of Maxwell's equations.

5. 2.1 in Thomson.

6. 2.3 in Thomson.

7. 2.4 in Thomson.

8. 2.6 in Thomson.

9. 2.8 in Thomson. Also, there is a typo in the process – explain what it should be.

10. 2.12 in Thomson.