4/8/13 Lecture outline

⋆ Reading: Zwiebach chapters 1 and 2.

• Metric convention (sigh..) $x^\mu = (ct, x, y, z)$, $x_\mu = (-ct, x, y, z) = \eta_{\mu\nu} x^\mu$, $\eta_{\mu\nu}\eta^{\nu\lambda} = \delta_\mu^\lambda$. Define $ds^2 = -dx^\mu dx_\mu$.

• Δs^2 for time-like, light-like, space-like separated events. Statement of causality principle: cause's effects only in the time-like future.

• Statement of relativity principle: physics is indistinguishable among all inertial frames. If one frame is inertial, the other inertial frames have linearly related coordinates, $x^{\mu'} = \Lambda^{\mu'}{}_\nu x^\nu$, where the transformations must preserve $ds^2 = 0$; that is enough to show that they preserve any $ds^2 = ds'^2$; that is enough to show that they preserve all 4-scalar products. This restricts the Lorentz transformations: if we write $\eta_{\mu\nu}$ as a matrix, the Lorentz transformations satisfy $\eta = \Lambda^T \eta \Lambda$. The Lorentz transformations consist of rotations and boosts (for a total of 3+3=6 independent generators). For the case of boosts, e.g. $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$.

Aside on the Lorentz transformations (question from lecture): writing transformation in matrix notation, need to account for upper vs lower indices, e.g. $\eta^{\mu\nu}$ vs $\eta_{\mu\nu}$.

• Light cone coordinates: $x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x^1)$. The bad: spoils rotational symmetry. The good: will make it easier to quantize string theory (avoids having to discuss here more advanced alternatives, which do not require going to the light cone coordinates). $-ds^2 = -2dx^+dx^- + dx_2^2 + dx_3^2 = -\widehat{\eta}_{\mu\nu} dx^\mu dx^\nu$. $a_\pm = -a^\mp$.

• $p^\mu = (E/c, p_x, p_y, p_z)$, with $p_\mu p^\mu = -m^2c^2$. p^μ transforms as a Lorentz 4-vector, $p^{\mu'} = \Lambda_\nu^{\mu'} p^\nu$. Proper time: $ds^2 = c^2 dt_p^2 = c^2 dt^2(1-\beta^2)$. $u^\mu = c dx^\mu/ds = dx^\mu/dt_p = \gamma(c, \vec{v})$, and $u_\mu u^\mu = -c^2$. A massive point particle has $p^\mu = m u^\mu$. Massless particles, like the photon, have p^μ with $p^\mu p_\mu = 0$.

• Quantum mechanics: replace $p^\mu = (H/c, \vec{p}) \to -i\hbar\partial^\mu$. Free particle wavefunction $\psi \sim \exp(ip \cdot x/\hbar)$; $p_\mu x^\mu \equiv p \cdot x$ is Lorentz invariant.

• Light cone coordinates: take $p^\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^1) = -p_\mp$. So $i\hbar\partial_{x^+} \to -p_+ = E_{lc}/c$, i.e. $p^- = E_{lc}/c$.