5/2/12 Lecture outline

⋆ Reading: Zwiebach chapter 7, 8, 9.

• Last time: pick gauge

$\tau = t$, define $ds \equiv |d\vec{X}|_{t=const}$, and choose σ such that

$$\partial_\sigma \vec{X} \cdot \partial_\tau \vec{X} = 0 \qquad \text{and} \qquad d\sigma = \frac{ds}{\sqrt{1 - v_\perp^2/c^2}} = \frac{dE}{T_0}.$$

(Using $H = \int T_0 ds/\sqrt{1 - v_\perp^2/c^2}$ and $\partial_t(ds/\sqrt{1 - v_\perp^2/c^2}) = 0$.) The last equation above is equivalent to $(\partial_\sigma \vec{X})^2 + c^{-2}(\partial_t \vec{X})^2 = 1$. Summarizing, we have

$$\dot{X} \cdot X' = 0 \qquad \dot{X}^2 + X'^2 = 0. \tag{1}$$

With this worldsheet gauge choice,

$$\mathcal{P}^{\tau\mu} = \frac{T_0}{c^2}\partial_t X^\mu = \frac{T^0}{c^2}(c, \vec{v}_\perp), \qquad \mathcal{P}^{\sigma,\mu} = -T_0 \partial_\sigma X^\mu = (0, -T_0 \partial_\sigma \vec{X}).$$

We can write this as

$$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'}\dot{X}^\mu \qquad \mathcal{P}^{\sigma\mu} = -\frac{c^2}{2\pi\alpha'}X^{\mu\prime}, \tag{2}$$

and then the EOM is simply a wave equation:

$$(\partial_\tau^2 - c^2\partial_\sigma^2)X^\mu = 0. \tag{3}$$

• Solution of the EOM for open string with free BCs: $\vec{X}(t,\sigma) = \frac{1}{2}(\vec{F}(ct+\sigma) + \vec{F}(ct-\sigma))$ where the open string has $\sigma \in [0, \sigma_1]$ and (1) implies that $|\frac{d\vec{F}(u)}{du}|^2 = 1$, and $\vec{X}'|_{ends} = 0$ implies $\vec{F}(u + 2\sigma_1) = \vec{F}(u) + 2\sigma_1 \vec{v}_0/c$. Note $\vec{F}(u)$ is the position of the $\sigma = 0$ end at time u/c. Then show that $\vec{v}_0$ is the average velocity of any point σ on the string over time interval $2\sigma_1/c$. Example from book: $\vec{X}(t, \sigma = 0) = \frac{\ell}{2}(\cos\omega t, \sin\omega t)$. Find $\vec{F}(u) = \frac{\sigma_1}{\pi}(\cos\pi u/\sigma_1, \sin\pi u/\sigma_1)$, giving $\vec{X}(t,\sigma) = \frac{\sigma_1}{\pi}\cos(\pi\sigma/\sigma_1)(\cos(\pi ct/\sigma_1), \sin(\pi ct/\sigma_1))$.