3/28/11 Lecture outline

• Introduction to gravity. Consider "mass" in three famous equations. Mass is inertia: $\vec{F} = m\vec{a}$; and it's gravitational "charge:" $\vec{F} = -Gm_1m_2\hat{r}/r^2$; and it's related to energy: $E = mc^2$. Why are they related? Does energy gravitate?

Compare gravity to electricity: $V_G = -Gm_1m_2/r$ vs $V_E = q_1q_2/r$. Both are long-range forces, massless force carrier. Information about charges propagate at speed of light, e.g. if a charge initially at rest is suddenly accelerated, there is a kink in the electric field lines that propagates at $v = c$. Gravity must behave the same: if a mass is suddenly moved, the gravitational field lines must have a gravitational wave kink, propagating at $v = c$. Such gravitational waves should carry energy and momentum. Indirectly observed in slow-down of orbiting binary pulsars (Nobel prize in 1993).

Newtonian gravity and special relativity clash. Einstein resolved the clash with the general theory of relativity in 1916. The slogan of special relativity is that physics in all inertial frames is equivalent. The slogan of general relativity is the equivalence principle: a local observer in free-fall can't detect gravity, in a way there is no gravity – it's replaced with space-time curvature, which is caused by mass or energy.

Compare and contrast GR and E&M. Charges can be positive or negative, electric fields tend to be screened; masses always positive, attraction of gravity is not screened. Both rest on a symmetry principle: gauge invariance in the case of E&M, and general coordinate covariance in the case of GR (details to follow). E&M is linear, whereas GR is nonlinear.

• Figure 1.1 from Hartle. Need full power of GR to understand relativistic stars, cosmology at the Universe, even the GPS system has to include GR effects.

• Review SR. Inertial frame with coordinate system $x^\mu = (ct, \vec{x})$. Notion of 4-vectors and the invariant interval: let's say that two events are separated in space-time by $ds^2 = -c^2dt^2 + d\vec{x}^2$. Similar to how we measure distance in space, except that time has the opposite signature. In another inertial frame of reference, coordinates $x^{\mu'} = (ct', \vec{x}')$, and the same two events are separated by $ds^{2'} = -cdt^{'2} + d\vec{x}^{'2}$. If two events are on light's trajectory, then $ds^2 = ds'^2 = 0$. Now can argue that $ds^2 = ds'^2$ for any two events: $ds^2 = a(|\vec{v}_{rel}|)ds'^2$, and with 3-frames would need $a(v_{12})a(v_{23}) = a(v_{13})$, which needs $a = 1$. Events are lightlike, spacelike, timelike separated for $ds^2 = 0, > 0, < 0$.

Four vectors $a^\mu = (a, \vec{a})$ and $b^\mu = (b, \vec{b})$, with dot product $a \cdot b = -a_0b_0 + \vec{a} \cdot \vec{b} \equiv a_\mu b^\mu$, where $a_\mu \equiv \eta_{\mu,\nu}a^\mu = (-a_0, \vec{b})$. Here $\eta_{\mu\nu} \equiv diag(-1, 1, 1, 1)$ is the fixed metric of SR.

GR will replace $\eta_{\mu\nu}$ with a dynamical metric $g_{\mu\nu}(x)$. This will be analogous to A_μ in E&M. The analog of Maxwell equations will be Einstein's equations, relating derivatives of the metric to the "charge" source of gravity: energy and momentum.

Inertial frames are related by $x^{\mu'} = \Lambda^{\mu'}{}_{\nu} x^\nu$. The dot product is preserved as long as $\eta_{\rho\sigma} = \Lambda^{\mu'}_{\rho} \Lambda^{\nu'}_{\sigma} \eta_{\mu'\nu'}$. Examples: rotate in x, y plane $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$; boost along x axis, $\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cosh\phi & -\sinh\phi \\ -\sinh\phi & \cosh\phi \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}$. Consider the origin $x' = 0$ in the original frame, $x/t = v = \tanh\phi$, so $\sinh\phi = \gamma v$ and $\cosh\phi = \gamma \equiv 1/\sqrt{1 - v^2/c^2}$. Set $c = 1$ from now on.