

Physics 225, Homework 4, Due Wednesday May 18.

1. Hartle 9.3. (a) How are a proton's E and $|\vec{P}|$ related in the Schwarzschild geometry?
(b) What are the p^μ components in the Schwarzschild basis in terms of E and $|\vec{P}|$?
2. Hartle 9.7. Two particles fall radially in from infinity in the Schwarzschild geometry, one with $e = 1$ and the other with $e = 2$. A stationary observer at $r = 6M$ measures the speed of each when they pass by. How much faster is the 2nd particle than the first?
3. Hartle 9.8. A spaceship moves without power in a circular orbit around a black hole of mass M . The Schwarzschild radius of the orbit is $7M$.
(a) What is the period of the orbit as measured by an observer at infinity?
(b) What is the period of the orbit as measured by a clock in the spaceship?
4. Hartle 9.14. Kepler's law says that equal areas of the ellipse are swept out in equal times. Consider the area swept out by an orbiting geodesic in the Schwarzschild geometry at some $r > 2M$. Does Kepler's area law hold using either proper time or Schwarzschild time?
5. Hartle 12.4. Consider the spacetime with

$$ds^2 = - \left(1 - \frac{M}{r}\right)^2 dt^2 + \left(1 - \frac{M}{r}\right)^{-2} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2).$$

(a) Find a transformation to new coordinates (v, r, θ, ϕ) , analogous to the Eddington-Finkelstein coordinates, that sets $g_{rr} = 0$ and shows that the geometry is not singular at $r = M$.

(b) Sketch a (\tilde{t}, r) diagram analogous to Fig. 12.2 showing world lines of ingoing and outgoing light rays and the light cones.

(c) Is this the geometry of a black hole?

6. Hartle 12.13. (a) An observer falls feet first into a Schwarzschild black hole, staring at her feet. Is there ever a moment when she cannot see her feet? Can she see her feet when her head is crossing the horizon? If so, at what radius does she see them? Does she ever see her feet hit the singularity at $r = 0$ assuming that she remains intact until her head reaches that radius? Analyze these questions with an Eddington-Finkelstein or Kruskal diagram.

(b) Is it dark inside a black hole? An observer outside sees a star collapsing to a black hole become dark. But would it be dark inside the horizon, assuming that the collapsing star continues to radiate at a steady rate as measured by an observer on its surface?

7. Hartle 12.22. Two observers in two rockets are hovering above a Schwarzschild black hole of mass M . They hover at fixed radius R such that

$$\left(\frac{R}{2M} - 1\right)^{1/2} e^{R/4M} = \frac{1}{2}$$

and fixed angular position (in fact, $R \approx 2.16M$). Observer A leaves this position at $t = 0$ and travels into the black hole on a straight line in a Kruskal diagram until destroyed in the singularity at the point where the singularity crosses the line $U = 0$. Observer B continues to hover at R .

- (a) Sketch the world lines of the two observers on a Kruskal diagram.
- (b) Is observer A following a timelike worldline?
- (c) What is the latest Schwarzschild time after observer A departs that B could send a light signal to A , in time for the light to reach A before A is destroyed by the singularity?