Physics 220, Lecture 7

⋆ Reference: Hamermesh.

• Last time: S_4 character table. Mention Young tableau for conjugacy classes (as before), and now also irreps.

• For general S_n, recall elements have k_j j-cycles and we define $\lambda_p = \sum_{j=p}^{n} k_j$, so the λ_p form an ordered partition of n. Draw Young tableaux with rows labeled by p and λ_p boxes in that row. Recall $n!/\prod_j j^{k_j} k_j!$ elements in that conjugacy class.

The tableaux is also associated with an irrep: put 1, ..., n in boxes and symmetrize horizontally and antisymmetrize vertically. The dimension of the irrep is $n!/H$ where H is the product of hooks factors. This result is a special case of Frobenius' formula for the characters of S_n (see Hamermesh).

• Recall last week, $L_0\psi = \lambda\psi$ has solutions $\psi_{a,K_a,i}$ and eigenvalues λ_{a,K_a} where $i = 1 \ldots n_a$ labels the basis of an irrep of symmetry group G. Suppose a perturbation breaks $G \to H$, which is some subgroup. Saw an example last week with masses on drumhead, breaking $O(2) \to D_4$. Then need to decompose G irreps into H irreps.

Example of how this is done, for $S_4 \to S_3$.

• Some symmetry group elements of molecules and crystals. An order n rotation axis, elements C_n^ℓ, $\ell = 1 \ldots n-1$, with $C_n^n = e$. Or label by rotation angle, $C(\phi)$. Reflection σ_h in plane perpendicular to axis; note $\sigma_h C(\phi)\sigma_h^{-1} = C(\phi)$, Reflection σ_v in plane passing through the axis; note $\sigma_v C(\phi)\sigma_v^{-1} = C(-\phi)$.

Law of rational indices for crystal faces. Choose any three non-coplanar edges of crystal and draw lines along them; these lines becomes 3 coordinates (u, v, w), and they meet at the origin of the coordinate system. If one crystal face is at (u, v, w) and another is at (u', v', w'), the observation is that $u'/u : v'/v : w'/w = n_1 : n_2 : n_3$. It can be shown (see Hamermesh) that this Implies $\cos(2\pi/n)$ must be a rational number, so $n = 1, 2, 3, 4, 6$ fold axes only.

Regular polyhedron: F faces, each being a polygon with s sides. At each vertex, n edges come together, and there is an n-fold axis through the vertex. V vertices and E edges; note $E = nV/2$ (each edge has a vertex at two ends) and $E = Fs/2$ (each edge shared by two faces). Euler: $V - E + F = 2$. Only solutions are the tetrahedron $(n, s, F = 3, 3, 4)$, cube $(3, 4, 6)$, octahedron $(4, 3, 8)$, dodecahedron $(3, 5, 12)$, icosahedron $(5, 3, 20)$.

• Octahedral group $\sim S^4$. Once again: symmetries and classes, character table. Vector and axial vector irreps, 2d irrep"E". . Subgroup: rotations taking even to even and odd to odd vertices. Factor group $= Z_2$. S_3 subgroup and decomposition of S_4 to S_3 irreps again.

• $H = H_0 + H_1$, where $H_0|E^0_{ax}, a, i = 1 \ldots n_a\rangle = E^0_{a,x}|"\rangle$. H_0 commutes with symmetry group G, so $H_0 g = g H_0$. Schur implies $H_0 \propto \delta_{ij}\delta_{ab}$ doesn't mix irreps and all states in irrep have same $E_{0,x}$. Also Schur implies $\langle b, j_b|a, i_a\rangle \propto \delta_{ab}\delta_{ij}$. Now H_1 breaks G to subgroup H. Consider H_0 with symmetry $O \cong S_4$ and H_1 with symmetry subgroup $D_3 \cong S_3$. Splitting of levels.

Next time:

Example of T. Emit a γ. Electric dipole transition and selection rules for leading order. Quadrupole radiation.

Octahedral symmetry group (e.g. UF_6). Include inversion: make 4-body diagonals have arrows, and I flips all. 48 elements in 10 classes