Physics 220, Lecture 14

⋆ Reference: Georgi chapters 6-7, first half of ch 8 if time.

• Start $SU(3)$, in particular mention $SU(3)_F$ and the (u,d,s) quark model. Plot $(T_3 = Q+Y/2, T_8 = \sqrt{\frac{3}{2}}Y)$, where $Y = B+S$ is the "hypercharge" and S is the strangeness (strangely, the strange quark has strangeness -1). Spin $j=0$ mesons (pions and Kaons), spin $j=1$ mesons in the $\mathbf{8} \oplus \mathbf{1}$. Baryons eg $j=\frac{1}{2}$ in the $\underset{\sim}{8}$ and $j=\frac{3}{2}$ in the $\mathbf{10}$. Problems with the model, e.g. spin-statistics, solved by $SU(3)_C$. Also $J/\psi \sim c\bar{c}$ meson discovery; no $SU(4)_F$.

• Roots and weights. Cartan generators, $H_{i=1\dots r}$ Hermitian, mutually commuting, with $\mathrm{Tr}(H_i H_j) = k_D \delta_{ij}$. Eigenvectors $|\mu, x, D\rangle$ with eigenvalues $H_i \to \mu_i$, write as weight vectors μ. Here μ is the generalization of m in $SU(2)$.

In the adjoint rep, label basis with the generators: $|T_a\rangle$, with $T_a|T_b\rangle = if_{abc}|T_c\rangle = |[T_a, T_b]\rangle$. Inner product is $\langle T_a|T_b\rangle = \lambda^{-1}\mathrm{Tr}(T_a^\dagger T_b)$. So $H_i|H_j\rangle = 0$ and $H_i|E_\alpha\rangle = \alpha_i|E_\alpha\rangle$, i.e. $[H_i, E_\alpha] = \alpha_i E_\alpha$ and $\langle H_i|H_j\rangle = \delta_{ij}$ and $\langle E_\alpha|E_\beta\rangle = \delta_{\alpha\beta}$. Take adjoint of $[H_i, E_\alpha] = \alpha_i E_\alpha$ to see that $E_\alpha^\dagger = E_{-\alpha}$. Here α_i are the weights of the adjoint rep, which are called the roots. Now note that in a general irrep we have that $E_{\pm\alpha}$ acts on $|\mu, D\rangle$ to raise and lower the roots, $\mu \to \mu \pm \alpha$. In the adoint rep, $E_\alpha|E_{-\alpha}\rangle = \beta_i|H_i\rangle$, where introducing an inner product and norm shows that $\beta_i = \alpha_i$, so $[E_\alpha, E_{-\alpha}] = \alpha \cdot H$.

• Lots of $SU(2)$s: write $E^\pm = |\alpha|^{-1}E_{\pm\alpha}$ and $E_3 = |\alpha|^{-2}\alpha \cdot H$ to get an $SU(2)$ subalgebra.

• Illustrate in $SU(3)$

• $SU(3)$ fundamental rep, $T_a = \frac{1}{2}\lambda_a$, Gell-Mann matrices, with $\lambda_3 = diag(1,-1,0)$ and $\lambda_8 = \frac{1}{\sqrt{3}}(1,1,-2)$. (Normalize to $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$. Since a rep and conjugate rep have T_a and $-T_a^*$ respectively, note $\mathbf{3}$ and $\overline{\mathbf{3}}$ differ. $\lambda_{1,2}$ are Pauli matrices $\sigma_{1,2}$ in the $(1,2)$ components, $\lambda_{4,5}$ are $\sigma_{1,2}$ in the $(1,3)$ entries, and $\lambda_{6,7}$ are similar in the $(2,3)$ entries. So $E_{\pm1,0} = \frac{1}{\sqrt{2}}(T_1 \pm iT_2)$, $E_{\pm1/2,\pm\sqrt{3}/2} = \frac{1}{\sqrt{2}}(T_4 \pm iT_5)$, $E_{\mp1/2,\pm\sqrt{3}/2} = \frac{1}{\sqrt{2}}(T_6 \pm iT_7)$.