Physics 220, Spring 2010, Homework 4, Due May 17

1-5. Georgi exercises 3A, 4A, 5C, 6C, 7C.

3A. Use the highest weight decomposition to show that

$$\{j\} \otimes \{s\} = \oplus_{\ell=|s-j|}^{s+j} \{\ell\}.$$

You don't need to construct the precise linear combinations that appear in each irrep (with the Clebsch Gordon coeffs), but just show how the counting of states goes at each stage of the highest weight decomposition.

4A Consider an operator O_α for $x = 1, 2$ in the spin 1/2 rep of $SU(2)$, $[J_a, O_x] = O_y(\sigma_a)_{yx}/2$. Given $\langle \frac{3}{2}, -\frac{1}{2}, \alpha|O_1|1, -1, \beta\rangle = A$, find $\langle \frac{3}{2}, -\frac{3}{2}, \alpha|O_2|1, -1, \beta\rangle$.

5C Δ^{++}, Δ^{+}, Δ^{0}, and Δ^{-} are isospin 3/2 particles ($T_3 = 3/2, \ldots -3/2$ respectively) with baryon number 1. They are produced by strong interactions in π nucleon collisions. Compare the probability of producing $\pi^+ p \to \Delta^{++}$ with that of producing $\pi^- p \to \Delta^0$ (where p is a proton).

6C Consider the simple Lie algebra formed by the ten matrices, σ_a, $\sigma_a\tau_1$, $\sigma_a\tau_3$, τ_2, where σ_a and τ_a are Pauli matrices in orthogonal spaces. Take the cartan generators to be $H_1 = \sigma_3$ and $H_2 = \sigma_3\tau_3$. Find
(a) The weights of the 4d representation.
(b) The weights of the adjoint representation.

7C Show that λ_2, λ_5 and λ_7 generate an $SU(2) \subset SU(3)$. Each $SU(3)$ irrep breaks up into a sum of $SU(2)$ irreps. How does the rep generated by the Gell-Mann matrices transform under this $SU(2)$? How does the adjoint representation transform under the $SU(2)$?