Physics 220, Homework 2, Due April 19

1. Last week you worked out the conjugacy classes of the quaternion group: $K_0 = \{e\}$, $K_1 = \{-1\}$, $K_2 = \{i, -i\}$, $K_3 = \{j, -j\}$, $K_4 = \{k, -k\}$.

   a) Explicitly construct all possible 1d representations of the group.

   b) Construct the character table for the group, using the character orthogonality etc. relations to fill in the entries for the $n_a > 1$ dimensional irrep(s). (The entries for the 1d irreps follow from the results of part (a).)

2. The tensor product of two irreps can be decomposed in terms of irreps: $D_a \otimes D_b = \oplus_c N^c_{ab} D_c$. The character satisfies $\chi_{D_a \otimes D_b}(g) = \chi_{D_a}(g)\chi_{D_b}(g)$. Using this and your results from the previous problem, write out all of the tensor product decompositions for the quaternion group ($D_a \otimes D_b =$? for every irrep a and b).

3. Georgi 1E: find the group T of all the discrete rotations (include only proper rotations, no inversions) that leave a regular tetrahedron invariant, by labeling the four vertices and considering the rotations as permutations of the four vertices. In other words, T is an (invariant) subgroup of S_4, corresponding to the permutations of the 4 vertices under rotation. So write all of the T group elements out in the S_4 notation as in sect. 1.19 of Georgi, e.g. $e = (1)(2)(3)(4) \in T$, and $(123)(4) \in T$, (whereas the S_4 element (1234) is not in T). You needn't bother writing out the full group multiplication table; just the group elements, conjugacy classes, and character table. Hints: As a check that you got all of the group elements, remember that the subgroup satisfies $|T| = |S_4|/m$ for some integer m. Also, be careful with the conjugacy classes: what was a single conjugacy class in S_4 can split into more than one conjugacy class in T.

4. For the group T of the previous problem, work out all of the tensor product decompositions, among all irreps $D_a \otimes D_b =$? (using the characters).

1