4/24/08 Lecture 8 outline

• Last time: Suppose A is Hermitian, $A^\dagger = A$. Then $\langle a_i|A|a_i\rangle^* = a_i^*\langle a_i|a_i\rangle = \langle a_i|A^\dagger|a_i\rangle = a_i\langle a_i|a_i\rangle$, from which it follows that $a_i = a_i^*$; the eigenvalues of Hermitian operators are real.

Also, using $A - A^\dagger = 0$, get $0 = \langle a_i(A - A^\dagger)|a_j\rangle = (a_j - a_i)\langle a_i|a_j\rangle$, so $a_i \neq a_j$ implies that $\langle a_i|a_j\rangle = 0$; eigenvectors with different eigenvectors are orthogonal.

We can use the eigenvectors of a Hermitian operator to form a (complete) basis, with $\langle a_i|a_j\rangle = \delta_{ij}$ and $\sum_i |a_i\rangle\langle a_i| = 1$ (if there are many eigenvectors with the same eigenvalue, all have to be included in these sums). In this basis, $A = \sum_i a_i|a_i\rangle\langle a_i|$ corresponds to a diagonal matrix. This is the statement that A can be diagonalized by a similarity transformation, given by the matrix of eigenvectors.

• Work through example of $A = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$. Find eigenvalues $a = \pm 1$ and eigenstates $|a = \pm 1\rangle$. Verify explicitly the above relations.

• If $[A, B] = 0$, then A and B can be simultaneously diagonalized. If $[A, B] \neq 0$, then they can not. Soon: will show that $[A, B] \neq 0$ corresponds to uncertainty, $\Delta A \Delta B \neq 0$. For example, $[\hat{x}, \hat{p}] = i\hbar$ will lead to the Heisenberg Uncertainty Principle, $\Delta x \Delta p \geq \frac{1}{2}\hbar$.

• Define expectation values in state $|\psi\rangle$ by $\langle A\rangle \equiv \langle\psi|A|\psi\rangle$. If A is Hermitian, then $\langle A\rangle$ is real. Note also that $\langle A\rangle = \sum_i a_i|\langle a_i|\psi\rangle|^2$.

• Next time: interpret the above as saying that a_i are the possible measured values of observable A, and $|\langle a_i|\psi\rangle|^2$ are the probabilities of measuring the outcome a_i in the state $|\psi\rangle$.