4/10/08 Lecture 4 outline

• Balmer formula for hydrogen spectral lines, $\lambda^{-1} = R(n_f^{-2} - n_i^{-2})$, $R \approx 1.01 \times 10^{-7} m^{-1}$. Rutherford scattering suggests hydrogen structure: proton in center, with orbiting electron. Classically would radiate and spiral in, would get radiation of frequency given by $e^2/4\pi\epsilon_0 r^2 = m_e \omega_e^2 r$, so $\omega_e^2 = (e^2/4\pi\epsilon_0 m_e) r^{-3}$ would get bigger as electron spirals in. Bohr (1913): $\omega_\gamma \neq \omega_e$. Instead quantized energy levels, and $E_\gamma = E_i - E_f$. Get Balmer formula if $E_n = -2\pi\hbar c R/n^2$.

Bohr argues for this by postulating that $L = m_e v r = m_e \omega r^2$ is quantized, $L = n\hbar$. Use classical relations to express E, r, and ω in terms of L:

$$E = -\tfrac{1}{2} m_e \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 L^{-2}, \qquad r = \frac{L^2}{m}\left(\frac{e^2}{4\pi\epsilon_0}\right)^{-1}, \qquad \omega_e = \frac{m}{L^3}\left(\frac{e^2}{4\pi\epsilon_0}\right)^2.$$

Now setting $L = n\hbar$ and defining $\alpha \equiv e^2/4\pi\epsilon_0\hbar c \approx 1/137$ this gives

$$E = -\tfrac{1}{2} m_e c^2 \alpha^2 n^{-2}, \qquad r = n^2 \frac{\hbar c}{m_e c^2 \alpha} \equiv n^2 a_0, \qquad \omega_e = \frac{m_e c^2 \alpha^2}{\hbar n^3}.$$

Then $E_\gamma = E_i - E_f$ agrees with Balmer. Also, ground state radius is $a_0 \equiv \hbar c/m_e c^2 \alpha \approx 0.529 \times 10^{-10} m$ in agreement with observation.

Generalization to Bohr-Sommerfeld quantization: $\oint p dx = nh$.

Correspondence principle: $\lim_{n\to\infty}$ of quantum results should agree with classical physics. Classically, $\omega_{electron} = m(e^2/4\pi\epsilon_0)^2 L^{-3}$, vs $\omega_\gamma = (E_i - E_f)/\hbar = -(2\hbar)^{-1} m_e c^2 \alpha^2 (n_i^{-2} - n_f^{-2})$. Take $n_i = n$ and $n_f = n-1$, get $\omega_\gamma = m_e c^2 \alpha^2 (2n-1)/2\hbar n^2 (n-1)^2$, which agrees with $\omega_{electron}$ for $n \to \infty$.

• Stern-Gerlach and angular momentum quantization, 1922. Inhomogeneous magnetic field with magnetic moment $\vec{\mu}$ leads to force, e.g. $F_z = \mu_z \partial B_z/\partial z$. Quantization of L_z leads to quantization of μ_z and this fits the observation of where the atoms land.

• 1923 de Broglie suggests that **all** matter has a wave/particle nature, $E = \hbar\omega$ and $\vec{p} = \hbar k$ (so $\lambda = h/p$). E.g. a pellet with mass $1g$ and velocity $1cm/sec$ has $\lambda = 2\pi/k = h/p \approx 10^{-26} cm$. This is too small to notice for objects that we see directly, but becomes important for objects like electrons. Electron's wave nature was seen in 1927 by Davisson and Germer (Bell labs) and George Thomson. Electrons scatter preferentially in certain directions, fits with Bragg scattering, constructive interference if $2d\sin\theta = n\lambda$.

• Describe interference in terms of individual photons (or electrons or anything). Also, describe outcome from series of polarizing sheets in terms of individual photons. How?

Probabilities! Probability amplitude interpretation of electron wave. Instead of particle trajectory, $x(t)$, have probability amplitude, $\psi(x,t)$. Linear superposition of **probability amplitudes**, e.g. 2 slits: $\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$. The amplitudes add, but the probabilities do not. The probability that particle is found in region from x to $x + dx$ is $P(x,t)dx = |\psi(x,t)|^2 dx$. Probability $P \sim |\psi|^2$ exhibits interference, $P \neq P_1 + P_2$, because of the cross terms, $|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_2^* \psi_1$. E.g. $\psi_1 = e^{ipx_1/\hbar}$ and $\psi_2 = e^{ipx_2/\hbar}$, with $p = \hbar = h/\lambda$, get $|\psi_1 + \psi_2|^2 = 4\cos^2(\pi \Delta x/\lambda)$, so constructive interference if $\Delta x \equiv x_1 - x_2 = n\lambda$ and destructive if $x_1 - x_2 = (n + \frac{1}{2})\lambda$, ($n$ is an integer).