4/1/08 Lecture 1 outline

• Today's lecture starts with a review of classical waves and particles, and then chapter 3 of Shankar.

• Particles and waves in classical physics. Particles= localized bundle of energy and momentum: q and $\dot{q}$ (in the Lagrangian description, or q and p in the Hamiltonian description). Equation of motion, e.g.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$$

determines $q(t)$ given initial conditions $q(t_i)$ and $\dot{q}(t_i)$. (Note that EOM is invariant under $t \to -it$.) On the other hand, waves = disturbance spread over space, $\psi(\vec{r}, t)$, e.g. a wave on a string has displacement satisfying the 1d wave equation

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}.$$

The 3d wave equation, e.g. for excess pressure describing a a sound wave, is similarly

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}.$$

Note that these are **linear** equations, so solutions obey superposition. Maxwell's equations give the wave equation for the components $\vec{E}$ and $\vec{B}$, with $v = c$, the speed of light. An example of a solution is a superposition of traveling waves, e.g. $\psi(x,t) = f(x - vt) + g(x + vt)$. Consider plane wave solutions:

$$\psi(x,t) = A\exp(ikx - \omega t) \equiv Ae^{i\phi}$$

where $k \equiv 2\pi/\lambda$ and $\omega \equiv 2\pi/T$, and A is the amplitude. Wave travels at $v = \frac{dx}{dt} = \omega/k = \lambda/T$. Write the general traveling wave as Fourier integrals, with $k = 2\pi/\lambda$, $\omega = 2\pi/T$, related by $\omega = vk$, e.g.

$$\psi(x,t) = \int_{-\infty}^{\infty} \frac{dk}{2\pi}\left(A(k)e^{i(kx-\omega t)} + B(k)e^{i(kx+\omega t)}\right).$$

These solutions are appropriate for an infinitely long string. In that case, k is arbitrary. But when the string has finite length L, with ends tied down, we have the boundary condition $\psi(x = 0, t) = \psi(x = L, t) = 0$. In that case, k is quantized, $k = \pi n/L$, i.e. $\lambda = 2L/n$. Then the above integral is replaced with a sum over the integer n. In 3d write

$$\psi(\vec{r}, t) = Ae^{i(k\cdot r - \omega t)} \qquad \omega = v|\vec{k}|.$$

The intensity of a wave is related to $|\psi|^2$, which is how there can be interference. Examples from water, sound, light waves.

• Experiment with waves and particles (classical). Illustrate interference. Slits S_1 and S_2. With just S_1 open get intensity $I_1 = |\psi_1|^2$, and with just S_2 open get $I_2 = |\psi_2|^2$. Arrival of energy is smooth function of x and t. With both open get $I_{1+2} = |\psi_1 + \psi_2|^2 \neq I_1 + I_2$: interference pattern. Maxima when $\Delta\phi = 2\pi n$, and minima when $\Delta\phi = (2n+1)\pi$, with n integer. At x_{min}, opening extra other slit has the effect of reducing the energy flow there. Now contrast with particles, where classical physics suggests $I_{1+2} = I_1 + I_2$.