130a Homework 5, These are practice problems, you don't need to turn them in.

1. Evaluate the following quantities:
   (a) $\langle p|\widehat{x}^2|x\rangle$.
   (b) $\langle p|\widehat{p}^2|x\rangle$.
   (c) $\langle x|\widehat{p}\widehat{x}|p\rangle$
   (d) $\langle x'|\widehat{x}^2|x\rangle$.

2. Write the following in the bra-ket notation. Simplify the result as much as possible (evaluate it if possible). Here $\psi_n(x) \equiv \langle x|E_n\rangle$ are complete, discrete, set of energy eigenstates, with $E_n \neq E_m$ for $n \neq m$. Also, $\phi_n(p) \equiv \langle p|E_n\rangle$.
   (a) $\int dx \int \frac{dp}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}\phi_m(p)\psi_n^*(x)$.
   (b) $\sum_n \psi_n(x)\phi_n^*(p)$.
   (c) $\sum_n \int \frac{dp}{\sqrt{2\pi\hbar}} e^{-ipx'/\hbar}\psi_n(x)\phi_n^*(p)$.
   (d) $\int dp \langle x|\widehat{H}|p\rangle\phi_n(p)$.

3. A particle is in a box of size a. Suddenly the box expands to size $b > a$. What is the probability that the particle will be found in the groundstate of the new potential? What is the probability it will be found in the first excited state?

4. A particle of mass m in a box of length L has wavefunction $\psi(x, t = 0) = \frac{1}{\sqrt{2}}u_1(x) + \frac{1}{\sqrt{3}}u_5(x) + \frac{1}{\sqrt{6}}u_7(x)$.
   (a) Write out $\psi(x, t)$ (in terms of the $u_n(x)$).

(b) What energies can be measured, and with what probabilities? Do these depend on time?

(c) What is the expectation value of the energy $\langle E\rangle$? Does this depend on time?

(d) At time $t = t_1$, the energy is measured. The result of the measurement is the largest of all the possible energies. What is the wavefunction immediately after the measurement?

(e) What is the wavefunction, as a function of t, for times after $t = t_1$? What energies can be measured, and with what probabilities, at a later time $t_2 > t_1$?