130a Homework 2, due 4/17

1. A crystal is a lattice array of atoms, with separation a. An incident particle can scatter off of one or another neighboring atom. The difference in the path length between the two scattered paths is $2a\cos\theta$, where θ is the angle of the incident electron, relative to the normal to the surface of the crystal. As observed by Davisson and Germer, and Thomson, the particle scatters constructively when the difference in path length is a whole number of wavelengths.

$$n\lambda = 2a\cos\theta.$$

Suppose that the crystal has $a = 4\mathring{A}$. What order of magnitude of kinetic energy of the incident particle would be needed to observe up to three interference maxima when the scattered particles are

(a) electrons (with $m_e c^2 \approx 0.51 MeV$)

(b) helium nuclei (with $m \approx 4GeV$).

Write the kinetic energies in units of eV.

2. Recall the harmonic oscillator which you met in problem 1 of last week's HW.

(a) Write the result of the Bohr Sommerfeld Wilson (BSW) quantization rule, $\oint pdx = nh$, when applied to one period of the classical motion. Here's what to do: plug in the classical solution (from last week's HW) into the BSW formula, and find the resulting quantization condition on the amplitude of the oscillation, $A_{classical} \to A_n = \ldots$.

(b) Combine the above result about A_n with the classical formula for the total energy of the oscillator, E, to find what BSW would have said are the quantized energy levels E_n of the harmonic oscillator. [Aside, we will see later do a proper quantum treatment of this problem, and will find that the BSW result is very nearly correct.]

(c) Suppose the harmonic oscillator goes from E_{n+1} to E_n by emitting a photon. According to the correspondence principle, the frequency ω_γ of the photon should agree with the frequency $\omega_{osc} = \sqrt{\kappa/m}$ of the harmonic oscillator, at least in the $n \to \infty$ limit. Show that the correspondence principle is, for this particular problem, actually satisfied for all n, without the need of $n \to \infty$.

3. A classical plane rotor has energy $E = L^2/2I$.

(a) Use BSW quantization to obtain the quantized energy levels E_n of a rotor.

(b) Verify the correspondence principle is satisfied when the transition is from level $n+1$ to n.

4. Verify that the same Bohr quantization for the hydrogen atom can be re-derived by requiring that the electron fits into a circular orbit with an integer number of wavelengths.

5. find the phase and group velocities for water waves in

   (a) shallow water, where $\omega = (2\pi)^{3/2}\sqrt{T/\rho\lambda^3}$ (where T is the surface tension and ρ is the density).

   (b) deep water, where $\omega = \sqrt{2\pi g/\lambda}$.

6. Consider a wave function of the form

$$\psi(x) = Ae^{-\mu|x|}.$$

   (a) Calculate the wavefunction $\phi(p)$ in momentum space.

   (b) Calculate A so that $\psi(x)$ is properly normalized.