Midterm, May 8 2007

Unless otherwise indicated, each part is worth 5 points, for a total of 100 points. Be sure to show your work. Just writing an answer, without showing how you got it, is NOT good enough. Also, some advice: the are many parts, and almost all can be worked out completely independently of each other. If you like, for example, you could work out part 2f first, before doing parts a through e. Work out the parts in whatever order you like. Do not spend too much time on any one problem, or part of the problem, at the expense of attempting the others!

$$\hbar c \approx 2000 eV \mathring{A} \qquad m_e c^2 \approx 0.5 \times MeV, \qquad e^2/4\pi\epsilon_0 \approx 1/137.$$

$$\int_{-\infty}^{\infty} dx e^{-\lambda x^2} = \sqrt{\frac{\pi}{\lambda}}, \qquad \int_{-\infty}^{\infty} dx\ x^2 e^{-\lambda x^2} = \tfrac{1}{2}\lambda^{-1}\sqrt{\frac{\pi}{\lambda}},$$

$$\int_{-\infty}^{\infty} dx\ x^{2n} e^{-\lambda x^2} = (-1)^n \frac{d^n}{d\lambda^n}\sqrt{\frac{\pi}{\lambda}}, \qquad \int_{-\infty}^{\infty} dx\ e^{bx} e^{-\lambda x^2} = e^{b^2/4\lambda}\sqrt{\frac{\pi}{\lambda}}.$$

$$\psi(x) = \int_{-\infty}^{\infty} \frac{dp}{\sqrt{2\pi\hbar}} \phi(p) e^{ipx/\hbar}, \qquad \phi(p) = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi\hbar}} \psi(x) e^{-ipx/\hbar}.$$

$$\rho = \psi^* \psi \qquad J = \frac{\hbar}{2im}\left(\psi^* \frac{d\psi}{dx} - \frac{d\psi^*}{dx}\psi\right), \qquad \frac{\partial \rho}{\partial t} + \frac{\partial J}{\partial x} = 0.$$

$$\text{wave modes:} \quad V\frac{d^3k}{(2\pi)^3}, \qquad \text{with spherical symmetry,} \quad V\frac{4\pi k^2 dk}{(2\pi)^3}.$$

$$\text{blackbody spectrum energy density} \qquad u(\nu, T) = \frac{8\pi h \nu^3}{c^3}\frac{1}{e^{h\nu/k_B T} - 1}.$$

$$\oint p dq = nh, \qquad \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta).$$

1. A certain metal has work function $6eV$. When light of a certain wavelength λ_0 is incident on the metal, the stopping voltage for the ejected electrons is 14 Volts. What is the numerical value of $2\pi/\lambda_0$ (in units of $\mathring{A}^{-1}$)?

2. Consider a particle of mass m, which moves in 1 dimension (the x direction), with potential $V(x) = \frac{1}{2}m\omega^2x^2$. Suppose that the particle is initially in a state with wavefunction $\psi(x,t=0) = \psi_0(x) \equiv C_0e^{-\alpha_0 x^2}$.

(a) Determine the correct value of C_0 (in terms of α_0) for this to lead to a properly normalized probability distribution. (A good check: verify that the units work).

(b). Verify that $\psi_0(x)$is a state of definite energy (a energy eigenvector), provided that α_0 takes a particular value. Determine this value of α_0 (in terms of m, ω, and $\hbar$). Show all your work for full credit.

(c) What is the energy eigenvalue E_0 corresponding to the above state $\psi_0(x)$ (assuming the particular value of α_0 determined above)?

(d) Suppose that the initial wavefunction is $\psi(x,t=0) = \psi_0(x)$. What is $\psi(x,t>0)$?

(e) Compute $\langle x\rangle$ and $\langle x^2\rangle$. If they're time dependent, compute them as a function of time (or, if you believe that they do not depend on time, write "time independent"). (It is OK to write your answers in terms of C_0 and α_0; no points deducted for that.) [10 points]

(f) Compute $\langle p\rangle$ and $\langle p^2\rangle$ (compute them in position space). [10 points]

(g) What is the probability that the particle is found to have momentum in the range from p to $p+dp$? Compute it directly, showing your work.

3. Again, consider a particle of mass m, with potential $V(x) = \frac{1}{2}m\omega^2x^2$. For this problem, you can freely make use of the following facts (which you do NOT need to derive here).

Fact 1: $\psi_0(x) \equiv C_0e^{-\alpha_0 x^2}$ is an energy eigenvector, with eigenvalue E_0.

Fact 2: $\psi_1(x) \equiv C_1xe^{-\alpha_0 x^2}$ is an energy eigenvector, with eigenvalue E_1.

Fact 3: they satisfy $\int_{-\infty}^{\infty} dx\ \psi_n^*(x)\psi_m(x) = \delta_{nm}$.

Important: You do **NOT** need to know what C_0, C_1, α_0, E_0, or E_1 are to answer the following questions. Just write your answers in terms of the ψ_n, and these various quantities.

Initial condition: $\psi(x,t=0) = \frac{1}{\sqrt{3}}e^{i\pi/3}\left(\psi_0(x) + i\sqrt{2}\psi_1(x)\right)$.

(a) Write down the wavefunction $\psi(x,t)$ for later times $t>0$.

(b) What is the probability that an energy measurement will give result E_0, and what is the probability that an energy measurement will give result E_1? Write these probabilities as a function of time, if they depend on time (or, if you believe that they do not depend on time, write "time independent").

(c) What is the expected value of the energy, $\langle E \rangle$? Again, write it as a function of time (or write "time independent").

(d) Compute the expected value of the position, $\langle x \rangle$. Again, write it as a function of time (or write "time independent").

(e) At time $t = t_*$, the energy is measured. The result obtained is $E = E_1$. What is the wavefunction $\psi(x,t)$ for times $t > t_*$ after the measurement?

(f) What energies can be measured, and with what probabilities, if the energy is measured again at a time after t_1?

(g) Compute $\langle E \rangle$ and $\langle x \rangle$ for times $t > t_1$. Again, write them as a function of time (or write "time independent").

4. Consider the square potential barrier,

$$V(x) = \begin{cases} V_0 & \text{for } 0 < x < L \\ 0 & \text{otherwise} \end{cases} .$$

(a) Write the form of the solution of the Schrodinger equation, for a state of energy $E < V_0$, in all 3 regions (to the left of the barrier, inside the barrier, and to the right of the barrier). Be sure to include both the spatial **and the time dependence** in your solution. Write your solution corresponding to a flux from left to right. As usual, write it where some terms have coefficient 1, and R, and T (reflection and transmission). You don't need to solve for these coefficients for this part, but you do need to make sure that the Schrodinger equation is solved in each region. [10 points]

(b) Write down the various equations needed to solve for the coefficients R and T. You don't need to solve the equations, just write them all out.