4/26/07 Lecture 8 outline

• Example: free particle in a box, between $x = 0$ and $x = L$. Energy eigenvectors $u_n(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$, with eigenvalue $E_n = \hbar^2\pi^2 n^2/2mL^2$, for $n = 1, 2, 3, \ldots$. Note groundstate energy $E_1 \neq 0$, connect to uncertainty principle.

These are the possible outcomes of experiments if the energy is measured. Suppose that the system is in the state $\psi(x) = u_n(x)$. Measurement of the energy will then yield the corresponding E_n with 100% probability. On the other hand, the position is uncertain. E.g. the probability to measure in range from x_1 to x_2 is

$$\int_{x_1}^{x_2} |u_n(x)|^2 dx = \frac{x_2 - x_1}{L} - \frac{1}{2\pi n}\left(\sin(\frac{2\pi n x_2}{L}) - \sin(\frac{2\pi n x_1}{L})\right).$$

First term is what we'd expect on average classically. The second term goes away for $n \to \infty$, which is an example of the correspondence principle. It is straightforward to work out $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, $\langle p^2\rangle$ etc.. You will check these and the uncertainty principle in a HW problem.

The u_n form an orthonormal and complete basis of functions with the correct BCs: any wavefunction can be expanded as

$$\psi(x) = \sum_{n=1}^{\infty} A_n u_n(x) \qquad u_n \equiv \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L},$$

where

$$A_n = \int_0^L dx\ u_n(x)^*\psi(x).$$

Here is the physics: if one measures the energy, the probability of measuring E_n is $|A_n|^2$. If the wavefunction is properly normalized, $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$, then the above expression for A_n will imply that $\sum_n |A_n|^2 = 1$, so the energy probabilities properly sum to unity. The average expectation value of the energy is

$$\langle E\rangle = \sum_n E_n|A_n|^2 = \langle H\rangle = \int_{-\infty}^{\infty} \psi^*(x)H\psi(x).$$

After measuring energy E_n, the state of the system is changed: now it is fully in the corresponding eigenstate, $\psi(x) = u_n(x)$. Subsequent measurement of energy will give the same E_n, with 100% probability.

• This is a very important and general aspect of quantum mechanics, true for any observable: the initial wavefunction is in a superposition of eigenvectors of the observable.

The coefficients of the superposition give the probability of measuring the corresponding eigenvalue. The measurement changes the state of the system: immediately afterward a measurement, the state is not in the superposition anymore, but rather it is fully in the eigenstate corresponding to the measured eigenvalue.

• The story of Schrodinger's cat, and its possible happy or sad ending. Some philosophy. For lack of time, we'll unfortunately only have time to explore the "shut-up and calculate" interpretation of QM.