4/10/07 Lecture 3 outline

• Finish reviewing history. Balmer formula for hydrogen spectral lines, $\lambda^{-1} = R(n_f^{-2} - n_i^{-2})$, $R \approx 1.01 \times 10^{-7} m^{-1}$. Rutherford scattering suggests hydrogen structure: proton in center, with orbiting electron. Classically would radiate and spiral in, would get radiation of frequency given by $e^2/4\pi\epsilon_0 r^2 = m_e\omega_e^2 r$, so $\omega_e^2 = (e^2/4\pi\epsilon_0 m_e)r^{-3}$ would get bigger as electron spirals in. Bohr (1913): $\omega_\gamma \neq \omega_e$. Instead quantized energy levels, and $E_\gamma = E_i - E_f$. Get Balmer formula if $E_n = -2\pi\hbar c R/n^2$. Bohr argues for this by postulating that $L = m_e v r = m_e \omega r^2$ is quantized, $L = n\hbar$. Then $E = \frac{1}{2}mv^2 - e^2/4\pi\epsilon_0 r = -e^2/8\pi\epsilon_0 r = -\frac{1}{2}m_e(e^2/4\pi\epsilon_0)^2 L^{-2}$ agrees with Balmer. Also can write $L^2 = m_e e^2 r/4\pi\epsilon_0$ to get an expression for the size of the atom, $r_n = n^2 a_0$, where a_0 is the groundstate radius, $a_0 = \hbar c/m_e c^2\alpha$, where $\alpha \equiv e^2/4\pi\epsilon_0\hbar c \approx 1/137$, gives $a_0 \approx 0.529 \times 10^{-10} m$ in agreement with observation.

Generalization to Bohr-Sommerfeld quantization: $\oint p dx = nh$.

Correspondence principle: $\lim_{n\to\infty}$ of quantum results should agree with classical physics. Classical result is found from $e^2/4\pi\epsilon_0 r^2 = m_e\omega_e^2 r$ and $L = m_e\omega r^2$ so $L^3 = m_e^3\omega_e^{-1}(e^2/4\pi\epsilon_0 m_e)^2$. Writing $L = n\hbar$, this gives $\omega_e = m_e c^2\alpha^2/\hbar n^3$. On the other hand, $E_i \to E_f$ transition gives a photon of frequency $\omega_\gamma = (E_i - E_f)/\hbar = -(2\hbar)^{-1} m_e c^2\alpha^2(n_i^{-2} - n_f^{-2})$. Take $n_i = n$ and $n_f = n-1$, get $\omega_\gamma = m_e c^2\alpha^2(2n-1)/2\hbar n^2(n-1)^2$, which agrees with ω_e for $n \to \infty$.

• Stern-Gerlach and angular momentum quantization, 1922. Inhomogeneous magnetic field with magnetic moment $\vec{\mu}$ leads to force, e.g. $F_z = \mu_z \partial B_z/\partial z$. Quantization of L_z leads to quantization of μ_z and this fits the observation of where the atoms hit the photographic plate.

• 1923 de Broglie suggests that matter (e.g. electrons) also have a wave/particle nature, with $\lambda = h/p$, i.e. $\vec{p} = \hbar\vec{k}$. Electron's wave nature was seen in 1927 by Davisson and Germer (Bell labs) and George Thomson. Electrons scatter preferentially in certain directions, fits with Bragg scattering, constructive interference if $2d\sin\theta = n\lambda$.

• Describe interference in terms of individual photons (or electrons or anything). Also, describe outcome from series of polarizing sheets in terms of individual photons. How? Probabilities! Probability amplitude interpretation of electron wave. Instead of particle trajectory, $x(t)$, have probability amplitude, $\psi(x,t)$. Linear superposition of **probability amplitudes**, e.g. 2 slits: $\psi(x,t) = \psi_1(x,t) + \psi_2(x,t)$. The amplitudes add, but the probabilities do not. The probability that particle is found in region from x to $x + dx$ is $P(x,t)dx = |\psi(x,t)|^2 dx$. Probability $P \sim |\psi|^2$ exhibits interference, $P \neq P_1 + P_2$, because of the cross terms, $|\psi_1 + \psi_2|^2 = |\psi_1|^2 + |\psi_2|^2 + \psi_1^* \psi_2 + \psi_2^* \psi_1$. E.g. $\psi_1 = e^{ipx_1/\hbar}$ and $\psi_2 = e^{ipx_2/\hbar}$, with $p = \hbar = h/\lambda$, get constructive interference if $x_1 - x_2 = n\lambda$ and destructive if $x_1 - x_2 = (n + \frac{1}{2})\lambda$, ($n$ is an integer).

• Free particle of mass m: momentum $\vec{p} = \hbar \vec{k}$ and energy $E = \vec{p}^2/2m = \hbar\omega$. Write probability amplitude as a plane wave

$$\psi(\vec{x}, t) \sim e^{i(\vec{p}\cdot\vec{x} - Et)/\hbar}.$$