4/5/07 Lecture 2 outline

•Last time: Planck's fix of the blackbody spectrum, assume $E = nh\nu$ quantized. Gives energy density of the glowing light in the cavity: $u(\nu, T) = 8\pi h\nu^3 c^{-3}(e^{h\nu/k_BT} - 1)^{-1}$. Fits beautifully the experimentally observed data, for $h = 6.6261 \times 10^{-34} J \cdot s$. For low frequencies, this agrees with the classical result. At high frequencies, it is very different, and avoids the divergent energy density. Also, emitted power per area per frequency $e(\omega, T) = cu(\omega, T)/4$. Integrating, get the Stephan-Boltzmann result for the total power per unit area $e_{total}(T) = \int_0^\infty e(\nu, T)d\nu = \sigma T^4$, with $\sigma = 2\pi^5 k_B^4/15c^2h^3$. These relations are very useful in astrophysics and cosmology.

The quantization of energy is more noticeable for larger frequencies. A natural unit of energy for atoms is the eV, $1eV = 1.602 \times 10^{-19}J$. In these units, $h \approx 4.13567 \times 10^{-15}eV/Hz$. As an example, optical frequencies are order few times $10^{14}Hz$, and the quanta of energy are of order $2eV$.

Einstein suggests that light is made up of particles, "photons" of energy $E = h\nu$. This also fits with a number of other experiments of the early 1900s, whose results could not be explained by the classical description of light as a wave.

• Photoelectric effect. Shine light on metal. Electrons kicked out. Measure maximum K.E. K_{max} via stopping voltage, $eV_s = K_{max}$. Find K_{max} doesn't depend on the brightness of the light, and there is no time lag. Not what classical physics would give. Instead, find K_{max} depends linearly on frequency ω. Suggests light as quanta of energy, photons, of energy $E = h\nu = \hbar\omega$. The ejected electron has kinetic energy $K_{max} = \hbar\omega - W$, where W is the work function, depends on the metal. Plot K_{max} vs ω, slope gives $\hbar$, agrees with Plank's. It works. In practice, $W \sim \mathcal{O}(1)eV$, with $1eV = 1.602 \times 10^{-19}J$.

• Compton effect. X-rays scatter off electrons just like scattering (relativistic) particles. Photons carry momenta $\vec{p} = \hbar\vec{k}$. Fits with $E = \hbar\omega$: then $E = pc$, since light travels with speed c, it must be massless. Scatter photons off electrons. Energy momentum conservation, $p_1 + p_2 = p_3 + p_4$. These are 4-vectors. We can write $p^\mu = (E/c, \vec{p})$ and recall that $p^2 = (E/c)^2 - \vec{p}^2 = (mc)^2$ in any frame of reference. Take $p_1 = (p, p, 0, 0)$, $p_2 = (m_ec, 0, 0, 0)$, $p_3 = (p', p'\cos\theta, p'\sin\theta, 0)$, and p_4 is the final momentum of the electron. Write $p_1 + p_2 - p_3 = p_4$ and square to get finally

$$\lambda' - \lambda = \lambda_c(1 - \cos\theta) \qquad \lambda_c = \frac{h}{m_ec} = 2.426 \times 10^{-12}m.$$

Change in wavelength is independent of intensity and time of exposure, depends only on scattering angle.