5/3/07 Lecture 10 outline

• Continue with 1d potentials. Draw qualitative pictures of various situations.

• Step well $V(x) = -V_0(\theta(x-a) - \theta(x+a))$. For $E > 0$, oscillating solutions in 3 regions, work out matching of coefficients by matching $u(x)'/u(x)$ across the boundaries. Find that there is no reflection when the distance $4a$ is an integer number of wavelengths: destructive interference of waves reflected at two edges (again, similar to optics).

• Potential barrier: $V(x) = V_0(\theta(x-a) - \theta(x+a))$. Consider $E < V_0$. Solution for $x < -a$ is $u = e^{ikx} + Re^{-ikx}$, for $|x| < a$ is $u = Ae^{-\kappa x} + Be^{+\kappa x}$ and for $x > a$ is $Te^{ikx}$. Matching gives

$$|T|^2 = \frac{(2k\kappa)^2}{(k^2+\kappa^2)^2 \sinh^2 2\kappa a + (2k\kappa)^2}.$$

For $\kappa a \gg 1$ get

$$|T|^2 \to \left(\frac{4\kappa k}{\kappa^2 + k^2}\right)^2 e^{-4ka}.$$

This illustrates tunneling.

More generally, $|T|^2 \sim \exp(-2\hbar^{-1} \int_{barrier} \sqrt{2m(V(x)-E)})$.

Example: decay of heavy elements by $\alpha$ tunneling in nucleus. Model the potential as

$$V(r) = \begin{cases} 0 & \text{if } r < r_0 \\ \frac{Z_1 Z_2}{r} & \text{if } r > r_0 \end{cases}$$

where $r_0$ is the radius of the nucleus, and $Z_1 = 2$ is the charge of the alpha particle (2 protons and 2 neutrons) and $Z_2 = Z - 2$ is the remaining charge of the nucleus. The energy $E$ of the emitted particles is such that $E < Z_1 Z_2 / r_0$. For example, $Z_1 = 2$, $Z_2 - 2 \approx 90$, and $r_0 \approx 10^{-15} m$ gives barrier height $Z_1 Z_2 / r_0 \approx 25$ MeV. But the emitted alpha particles only have $E$ of the order of 5 to 10 MeV. And yet they escape – thanks to tunneling.

• Bound states in potential well.