4/3/07 Lecture 1 outline

• An old question: is light a particle or a wave? In 1800 Young's double slit experiment showed interference, suggests light is a wave. Later understood as solution of Maxwell's equations. Double slit, intensity $I \sim |\vec{E}_1 + \vec{E}_2|^2 = 4I_1 \cos^2(k\Delta L/2)$. Also explains Snell's law, lenses, thin film interference, diffraction, diffraction gratings, etc. Visible light has $\lambda \sim 4 - 7 \times 10^{-7} m$.

• But it turns out that this, plus concepts from thermodynamics, leads to a paradox. Also, disagreement with experiments around 1900.

• First let's recall waves. First in 1d, waves on a string.

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}.$$

Because this is a linear equation, we can use superposition of solutions. An example of a solution is a superposition of traveling waves, e.g. $\psi(x,t) = f(x-vt) + g(x+vt)$. Write the general traveling wave as Fourier integrals, with $k = 2\pi/\lambda$, $\omega = 2\pi/T$, related by $\omega = vk$, e.g.

$$\psi(x,t) = \int_{-\infty}^{\infty} \frac{dk}{2\pi}\left(A(k)e^{i(kx-\omega t)} + B(k)e^{i(kx+\omega t)}\right).$$

These solutions are appropriate for an infinitely long string. In that case, k is arbitrary. But when the string has finite length L, with ends tied down, we have the boundary condition $\psi(x=0,t) = \psi(x=L,t) = 0$. In that case, k is quantized, $k = \pi n/L$, i.e. $\lambda = 2L/n$. Then the above integral is replaced with a sum over the integer n.

• For waves moving in 3d, we have $k = |\vec{k}|$, where $\vec{k}$ points in the direction that the wave is moving. Replace $\partial^2/\partial x^2 \to \nabla^2$ in wave equation. Solutions look like e.g.

$$\psi(\vec{x},t) = \int \frac{d^3k}{(2\pi)^3} A(\vec{k}) e^{i(\vec{k}\cdot\vec{x} - \omega t)},$$

with $\omega = vk$.

The intensity of a wave is related to $|\psi|^2$, which is how there can be interference. Examples from water, sound, light waves.

• Blackbody radiation and the UV catastrophe. Picture each $\vec{k}$ mode as a harmonic oscillator, one for each polarization. Let $N(\omega)d\omega$ be the number of wave modes in the frequency range from ω to $\omega + d\omega$. Consider first waves on a string, $\psi(x,t) = A\sin(kx)\cos(\omega t)$, $k = n\pi/L$, $n = 1, 2 \ldots$. The number of modes in interval Δk is $L\Delta k/\pi$, so $N(k)dk = Ldk/2\pi$, where the 2 is because the standing wave is a superposition of 2 traveling waves, with k and $-k$. So can replace $\sum_{n=1}^{\infty} \to \int_{-\infty}^{\infty} Ldk/2\pi$. For 3d waves, we replace $\sum_{n_x,n_y,n_z} \to \int Vd^3\vec{k}/(2\pi)^3$. Write in spherical coordinates, use $\omega = ck$, and recall there are 2 polarizations, to get $N(\omega)d\omega = V\omega^2 d\omega/\pi^2 c^3$. If we write $\omega = 2\pi\nu$ and $N(\omega)d\omega = N(\nu)d\nu$, this gives $N(\nu) = 8\pi V\nu^2/c^3$. Note that this density diverges for large frequencies.

Each $\vec{k}$ mode of light behaves as a harmonic oscillator. A classical harmonic oscillator, at temperature T, has average energy $k_B T$, independent of the frequency. The energy density in the range ω to $\omega + d\omega$ would then be $u_{cl}(\omega, T) = N(\omega)k_B T d\omega/V = k_B T\omega^2/\pi^2 c^3$. Crazy! For any $T \neq 0$, would have divergent energy density at large frequencies. UV catastrophe. A paradox in classical physics.

• Planck's fix: assume radiation of frequency ν can only be absorbed or emitted in quantized amounts, given by $E = nh\nu$ for integer n. In this case, the average energy of a harmonic oscillator of frequency ω is $\overline{E}(\omega, T) = \sum_{n=0}^{\infty} n\hbar\omega e^{-n\hbar\omega/k_B T} / \sum_{n=0}^{\infty} e^{-n\hbar\omega/k_B T} = \hbar\omega(e^{\hbar\omega/k_B T} - 1)^{-1}$. This result agrees with the classical answer of $k_B T$ for low frequencies, $\hbar\omega \ll k_B T$, but differs for large frequencies.

Leads to $u(\nu, T) = 8\pi h\nu^3 c^{-3}(e^{h\nu/k_B T} - 1)^{-1}$. Fits beautifully the experimentally observed data, for $h = 6.6261 \times 10^{-34} J \cdot s$. For low frequencies, this agrees with the classical result. At high frequencies, it is very different, and avoids the divergent energy density. Also, emitted power per area per frequency $e(\omega, T) = cu(\omega, T)/4$. Integrating, get the Stephan-Boltzmann result for the total power per unit area $e_{total}(T) = \int_0^{\infty} e(\nu, T)d\nu = \sigma T^4$, with $\sigma = 2\pi^5 k_B^4/15c^2 h^3$.