5/17/07 Homework 6 Due May 24, 2007.

1. Evaluate the following quantities:
   (a) $\langle p|\widehat{x}^2|x\rangle$.
   (b) $\langle p|\widehat{p}^2|x\rangle$.
   (c) $\langle x|\widehat{p}\widehat{x}|p\rangle$
   (d) $\langle x'|\widehat{x}^2|x\rangle$.

2. Write the following in the bra-ket notation. Simplify the result as much as possible (evaluate it if possible). Here $\psi_n(x) \equiv \langle x|E_n\rangle$ are complete, discrete, set of energy eigenstates, with $E_n \neq E_m$ for $n \neq m$. Also, $\phi_n(p) \equiv \langle p|E_n\rangle$.
   (a) $\int dx \int \frac{dp}{\sqrt{2\pi\hbar}} e^{ipx/\hbar}\phi_m(p)\psi_n^*(x)$.
   (b) $\sum_n \psi_n(x)\phi_n^*(p)$.
   (c) $\sum_n \int \frac{dp}{\sqrt{2\pi\hbar}} e^{-ipx'/\hbar}\psi_n(x)\phi_n^*(p)$.
   (d) $\int dp\langle x|\widehat{H}|p\rangle\phi_n(p)$.

3. The following problems are about the harmonic oscillator (to be discussed in lecture Tuesday). Once you understand the method, the following problems can be done very quickly. Use the notation $|n\rangle \equiv |E_n\rangle$, and $\psi_n(x) \equiv \langle x|n\rangle$. Evaluate the following, using the bra-ket notation, and creation and annihilation operators.
   (a) $\langle m|a^3|n\rangle$. (Write your answer in terms of a general n and m, indicating when it it is zero, and when it's non-zero.)
   (b) $\langle m|\widehat{x}|n\rangle$. (ditto)
   (c) Evaluate $\langle x\rangle$, $\langle p\rangle$, $\langle x^2\rangle$, and $\langle p^2\rangle$, in the state $\psi_n(x) \equiv \langle x|n\rangle$.
   (d) Evaluate $\langle n+1|\widehat{x}^3|n\rangle$.
   (e) Evaluate $\langle n+3|\widehat{p}^3|n\rangle$.