Midterm Cheat Sheet

$$dv = -u\frac{dm}{m}$$

$$d\vec{l} = \hat{r}dr + rd\theta\hat{\theta} + r\sin\theta d\phi\hat{\phi} = ds\hat{s} + sd\phi\hat{\phi} + dz\hat{z}.$$

$$\nabla F = \frac{\partial F}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial F}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial F}{\partial \phi}\hat{\phi} = \frac{\partial F}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial F}{\partial \phi}\hat{\phi} + \frac{\partial F}{\partial z}\hat{z}.$$

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi} = \frac{1}{s}\frac{\partial}{\partial s}(sF_s) + \frac{1}{s}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}.$$

$$df = \nabla F \cdot d\vec{l}, \qquad \int_V (\nabla \cdot \vec{F})dV = \oint_{\partial V} \vec{F} \cdot d\vec{a}, \qquad \int_S (\nabla \times \vec{F}) \cdot d\vec{a} = \oint_{\partial S} \vec{F} \cdot d\vec{l}.$$

$$\nabla \cdot (\frac{\hat{r}}{r^2}) = 4\pi\delta^3(\vec{r})$$

$$\vec{E}(\vec{r}) = \sum_i \frac{q_i(\vec{r} - \vec{r}_i)}{4\pi\epsilon_0|\vec{r} - \vec{r}_i|^3} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')(\vec{r} - \vec{r}')dV'}{|\vec{r} - \vec{r}'|^3}.$$

$$\nabla \cdot E = \rho/\epsilon_0, \qquad \nabla \times \vec{E} = 0, \qquad \vec{E} = -\nabla\phi.$$

$$\phi(\vec{r}) = -\int_O^{\vec{r}} \vec{E} \cdot d\vec{l} = \sum_i \frac{q_i}{4\pi\epsilon_0|\vec{r} - \vec{r}_i|} = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma(\vec{r}')da'}{|\vec{r} - \vec{r}'|}.$$

$$W_{1\to 2} = \int_1^2 (-Q\vec{E}) \cdot d\vec{l} = Q(\phi(\vec{r}_2) - \phi(\vec{r}_1)).$$

$$U = \tfrac{1}{2}\sum_i q_i\phi(\vec{r}_i) = \tfrac{1}{2}\int \rho\phi dV = \frac{\epsilon_0}{2}\left(\int_V \vec{E}^2 dV + \oint_{\partial V} \phi\vec{E} \cdot d\vec{a}\right) = \frac{\epsilon_0}{2}\int \vec{E}^2 dV.$$

$$(\vec{E}_{above} - \vec{E}_{below}) \cdot \hat{n} = -\frac{\partial V}{\partial n}|_{above} + \frac{\partial V}{\partial n}|_{below} = \sigma/\epsilon_0$$

$$\vec{p} = \frac{\sigma}{2}(\vec{E}_{above} + \vec{E}_{below}) = \frac{\sigma^2}{2\epsilon_0}\hat{n} = \frac{\epsilon_0}{2}\vec{E}^2\hat{n}$$