10/30/19 Lecture outline

• Last time:

$$\frac{dP}{T} = \frac{V}{\prod_i (2E_i V)} |\mathcal{A}_{fi}|^2 d\Pi_{LIPS}, \qquad d\Pi_{LIPS} \equiv (2\pi)^4 \delta^4(p_f - p_i) \prod_f \frac{1}{(2\pi)^3 (2E_f)} d^3 p_f$$

where $d\Pi_{LIPS}$ is the Lorentz invariant phase space for the final states. For two body final states (in CM frame): $D = \int d\Pi_{LIPS} = \int \frac{d^3\vec{p}_1}{(2\pi)^3 2E_1} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^3(\vec{p}_1 + \vec{p}_2)\delta(E_1 + E_2 - E_T)$

$$D = \int \frac{1}{(2\pi)^3 4E_1 E_2} p_1^2 dp_1 d\Omega_1 (2\pi)\delta(E_1 + E_2 - E_T).$$

Using $E_1 = \sqrt{p_1^2 + m_1^2}$ and $E_2 = \sqrt{p_1^2 + m_2^2}$ get $\partial(E_1 + E_2)/\partial p_1 = p_1 E_T / E_1 E_2$ and finally $D = p_1 d\Omega_1 / 16\pi^2 E_T$. This should be divided by 2! (i.e. $n_f!$) if the final states are identical.

• Summary:

$$d\Gamma_{1\to 2} = \frac{|\mathcal{A}|^2 D_{2-body}}{2M}$$

$$d\sigma_{2\to 2} = \frac{|\mathcal{A}|^2}{4E_1 E_2} D_{2-body} \frac{1}{|\vec{v}_1 - \vec{v}_2|}$$

$$D_{2-body(CM)} = \frac{p_1 d\Omega_1}{16\pi^2 E_{CM}} \qquad \text{(divide by 2! if identical final states).}$$

• Example. For $\mu^2 > 4m^2$, consider $\phi \to \bar{N}N$ decay in the toy model. $\mathcal{A} = -g + \mathcal{O}(g^3)$, and get

$$\Gamma = \frac{g^2}{2\mu} \frac{p_1}{16\pi^2 \mu} \int d\Omega_1 = \frac{g^2}{8\pi\mu^2} \frac{\sqrt{\mu^2 - 4m^2}}{2} + \mathcal{O}(g^4),$$

For $2 \to 2$ scattering in the CM frame,

$$d\sigma = \frac{|\mathcal{A}|^2}{4E_1 E_2} \frac{p_f d\Omega_1}{16\pi^2 E_T} \frac{1}{|\vec{v}_1 - \vec{v}_2|} = \frac{|\mathcal{A}|^2 p_f d\Omega_1}{64\pi^2 p_i E_T^2}$$

where we used $|\vec{v}_1 - \vec{v}_2| = p_1(E_1^{-1} + E_2^{-2}) = p_i E_T / E_1 E_2$ in the CM frame, and p_i is the magnitude of the initial 3-momentum, and p_f is that of the final momentum; they can be different if the initial and final states are of particles of different masses, e.g. $e^+e^- \to \mu^+\mu^-$.

• Let's now consider the theory with $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$, with real scalar field ϕ and λ is a real coupling constant that we will take to be small and treat in perturbation theory. The requirement that the potential $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$ be bounded below requires $\lambda \geq 0$. There is a $\mathbf{Z}_2$ symmetry $\phi \to -\phi$. For $m^2 > 0$, the potential has a single vacuum at $\phi = 0$. For $m^2 < 0$ there are two vacua at $\langle\phi\rangle$; this is an example of spontaneous (discrete) symmetry breaking, which will be discussed more later. We will take $m^2 > 0$.

Consider $\phi(p_1)+\phi(p_2) \to \phi(p_1')+\phi(p_2')$ scattering. The leading order amplitude is $\mathcal{A} = -\lambda+\mathcal{O}(\lambda^2)$. The associated Born Approximation potential is $V(\vec{r}) = -\frac{\lambda}{(2m)^2}\int \frac{d^3p}{(2\pi)^3}e^{i\vec{p}\cdot\vec{r}} = -\frac{\lambda}{(2m)^2}\delta^3(\vec{r})$. Comment about the combinatorics. Write down the Feynman rules.

Now consider the $\mathcal{O}(\lambda^2)$ correction to $2 \to 2$ scattering: $i\mathcal{A} \supset (-i\lambda)^2(F(s) + F(t) + F(u))$ where $F(p^2) \equiv \frac{1}{2}\int \frac{d^4k}{(2\pi)^4}\frac{i}{k^2-m^2+i\epsilon}\frac{i}{(k+p)^2-m^2+i\epsilon}$ where the $\frac{1}{2}$ is a symmetry factor. The integral is log divergent for large k and requires being regulated and renormalization; this will be discussed next quarter.

• Amplitudes are computed from Feynman diagrams upon amputating the external propagators and putting the external states on shell (imposing $p_i^2 = m_i^2$ for the initial and final states). It is also useful to consider the quantities without the external propagators amputated or on shell; these quantities are called Greens functions.

⋆ **Reading for the upcoming part: Coleman lecture notes pages 140-175.**

• Brief introduction to a better description of QFT and perturbation theory.]Define the true vacuum $|\Omega\rangle$ such that $H|\Omega\rangle = 0$, and $\langle\Omega|\Omega\rangle = 1$. The true vacuum of an interacting QFT is a complicated beast – it can be thought of roughly as a soup of particle-antiparticle states – it can not be solved for solved for exactly. (Progress: in classical mechanics, can solve 2 body problem exactly, but ≥ 3 body only approximately; in GR, can solve 1 body problem exactly, but ≥ 2 body only approximately; in QM can generally solve even only 1-body problem only approximately, but at least the 0-body problem is trivial; in QFT, even the 0-body problem is not exactly solvable.)

Define Green functions or correlation functions by

$$G^{(n)}(x_1, \ldots x_n) = \langle\Omega|T\phi_H(x_1)\ldots\phi_H(x_n)|\Omega\rangle,$$

where $\phi_H(x)$ are the full Heisenberg picture fields, using the full Hamiltonian.

Now show that

$$G^{(n)}(x_1 \ldots x_n) = \frac{\langle 0|T\phi_{1I}(x_1)\ldots\phi_{nI}(x_n)S|0\rangle}{\langle 0|S|0\rangle},$$

where $|0\rangle$ is the vacuum of the free theory, and ϕ_{iI} are interaction picture fields, and the S in the numerator and denominator gives the interaction-Hamiltonian time evolution from $-\infty$ to x_n, then from x_n to x_{n-1} etc and finally to $t = +\infty$. To show it, take $t_1 > t_2 \ldots > t_n$ and put in factors of $U_I(t_a, t_b) = T\exp(-i\int_{t_a}^{t_b} H_I)$ to convert ϕ_I to ϕ_H, using $\phi_H(x_i) = U_I(t_i, 0)^\dagger\phi_I(x_i)U_I(t_i, 0)$.