215a Homework exercises 5, Fall 2019, due Nov. 13

1. Question 2 on Tong's example sheet 4. Also for each diagram determine the symmetry factor (recall this is from the factors of 1/4! that accompany the λ and that are usually canceled by the 4! combinatoric choices of which vertex leg gets Wick contracted with something else, but if the internal propagator contractions have some permutation symmetry then there is not an independent 4! factor for each vertex, so some of the 1/4!s are incompletely canceled).

2. Consider the theory of a scalar field ϕ with $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3$.

(a) What is the leading order S-matrix $\langle\phi(p_2)\phi(p_3)|S|\phi(p_1)\rangle$ and corresponding amplitude for the process $\phi(p_1) \to \phi(p_2) + \phi(p_3)$? Draw the Feynman diagram and write out the Feynman rules in momentum space.

(b) Draw all of the Feynman diagrams for the same $\phi \to \phi\phi$ process at the next order in perturbation theory. Write down the mathematical expression for each of these diagrams (do not attempt to evaluate the integral, since they require a regulator and further discussion).

(c) Write down the position space Greens function $G^{(3)}(x_1, x_2, x_3)$ to order g^3 in perturbation theory, by considering Feynman diagrams in position space.

(d) Consider the $G^{(3)}(x_1, x_2, x_3)$ contribution from part (c) whose diagram consists of a circle with three attached legs. Fourier transform and apply LSZ to this contribution and verify that this gives a contribution to $\langle\phi(p_2)\phi(p_3)|S|\phi(p_1)\rangle$ that agrees with the corresponding momentum space Feynman diagram that you found in part (b).

3. Consider the quantum mechanical simple harmonic oscillator, with $L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}\omega^2 q^2$. Write down $q_H(t)$ in terms of the usual creation and annihilation operators and verify (a)

$$\langle 0|Tq_H(t_1)q_H(t_2)|0\rangle = -iG_{SHO}(t_2 - t_1),$$

$$G_{SHO}(t) = \int_{-\infty}^{\infty} \frac{dE}{2\pi} \frac{e^{-iEt}}{-E^2 + \omega^2 - i\epsilon} = \frac{i}{2\omega} e^{-i\omega|t|}.$$

(b)

$$\langle 0|Tq_H(t_1)q_H(t_2)q_H(t_3)q_H(t_4)|0\rangle = (-i)^2(G_{SHO}(t_2 - t_1)G_{SHO}(t_3 - t_4) + perms),$$

where perms denote similar terms with $t_2 \leftrightarrow t_3$ and $t_2 \leftrightarrow t_4$.