Physics 105a, Ken Intriligator lecture 2, October 3, 2017

• Just for fun, use mathematica to show π and e to many decimal places.

• Illustrate solving $Mx = a$ for given matrix M and vector a using both Inverse and Solve.

• Example: form matrix from too few linearly independent vectors and find its NullSpace.

• Illustrate = vs := by $x = RandomInteger[1, 1000]$ and $Table[x, 200]$ vs $x := RandomInteger[1, 1000]$ and $Table[x, 200]$.

• Example: $\sum_{j=1}^{n} j^{-k}$ and value for $n = \infty$ for all k (comment on $k = -1$).

• Consider a particle of mass m in 1d, with potential $V(x)$, get $m\frac{d^2x(t)}{dt^2} = -V'(x)$. Equilibrium where $V'(x) = 0$, suppose it is at $x = 0$. Then expand for small x as $V(x) = V_0 + \frac{1}{2}kx^2 + \mathcal{O}(x^3)$, and we again get the equation from last time. Example with mathematica, taking $V(x) = V_0(1 - \cos Cx)$, using Series$[V[x], \{x, 0, n\}]$ for various n and then $V_{approx} = Normal[Series[V[x], \{x, 0, 2\}]$.

Write solution as $x = Re[Ae^{i\omega t}]$ with A complex, so it has 2 real constants to correspond to the two constants of integration. Solve for A in terms of x_0 and v_0. Check solution with mathematica. Make a plot. Preview: $DSolve[\{x''[t] == -w^2 x[t], x[0] == x0, x'[0] == v0\}, x[t], t]$.

Illustrate ComplexExpand$[z]$.

• Let's make a short excursion into the complex plane. Defining functions by analytic continuation, e.g. e^z, $\sin z$, $\log z$. Illustrate Re, Im, Abs, Arg and $Conjugate$ in Mathematica. Analytic functions and the Cauchy Riemann equations. Plot in mathematica (using VectorPlot of the real and imaginary parts) e.g. $f(z) = z^n$ for various n, $f(z) = \bar{z}$ (not analytic), $z^{1/2}$, $z^{1/3}$. Also in mathematica compare e.g. Series$[z^{1/2}, \{z, z_0, 4\}]$ for $z_0 = 0$ vs $z_0 = 1$. The CR equations imply that $\Re f(z)$ and $\Im f(z)$ are solutions of the 2d Laplace equations; examples.

• Define poles and residues and cuts. Cauchy's theorem; explain why e.g. $\oint dz/z = 2\pi i$ (assuming the origin is encircled) vs $\oint dz z^n = 0$ for n any integer other than -1. For functions like $z^{1/2}$, with branch cuts, we need to either avoid the cut, or take care when crossing it, or sometimes it's useful to hug around either side of a cut and account for the difference. There are many applications to physics. We will only scratch the surface of these methods in this class.

• Example of evaluating integrals by contour integration. E.g. $\oint dz/z$, and state Cauchy's theorem. Example: $\int_0^\infty dx(1 + x^2)^{-1} = \pi/2$. Now check it with Mathematica.

- Gamma function $\Gamma(z)$; give integral definition and type it into mathematica, $\Gamma(z+1) = z\Gamma(z)$ and relation to factorial. Poles at $x = 0$ and negative integers. Check with mathematica. Also $\Gamma(z)\Gamma(1-z) = \pi/\sin(\pi z)$.
- Gaussian integral, including in multi-dimensions, and relation to spherical integrals and solid angles.