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Quantum Mechanics A (Physics 212A) Fall 2016
Worksheet 4

Announcements

- The 212A web site is:

  http://keni.ucsd.edu/f16/ .

  Please check it regularly! It contains relevant course information!

Problems

1. **Give it a Kick**

   Consider the $D = 1$ simple harmonic oscillator in its groundstate. Suppose something kicks the system imparting an additional momentum p_0. What's the probability the system remains in the ground state?

   (a) What's the new Hamiltonian for the system? Express this in terms of the usual ladder operators $\hat{a}$ and $\hat{a}^\dagger$

   (b) Define a new operator $\hat{A} \equiv \hat{a} - \beta$ where $\beta \equiv \frac{1}{\mathbf{i}\omega} \frac{p_0}{m} \sqrt{\frac{m\omega}{2}}$.
   Show that the $\hat{A}$ are ladder operators: $[\hat{A}, \hat{A}^\dagger] = 1$

   (c) Rewrite the new Hamiltonian in terms of these operators, what do you find?

   (d) Relate the original groundstate $|0\rangle$ to the new groundstate $|\beta\rangle$

   (e) Using $|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle$ compute $P = |\langle 0|\beta\rangle|^2$
   Hint: Insert identity and use the relation above.

2. **Bogliubov Transformation**

   We solved a new Hamiltonian by defining a set of transformed creation/annihilation operators which satisfy the same algebra $[A, A^\dagger] = \mathbb{1}$

   More generally consider $\hat{b} = \hat{a}\cosh\eta + \hat{a}^\dagger \sinh\eta$

   (a) Show that $[\hat{b}, \hat{b}^\dagger] = \mathbb{1}$

   (b) Show that $\hat{b} = U\hat{a}U^\dagger$ for $U = e^{\frac{\eta}{2}(\hat{a}\hat{a} - \hat{a}^\dagger\hat{a}^\dagger)}$

   (c) Show for fermionic operators $\hat{c}^2 = 0 = (\hat{c}^\dagger)^2$ and $\{\hat{c}, \hat{c}^\dagger\} = \mathbb{1}$ that
   $\hat{d} = \hat{c}\cos\theta + \hat{c}^\dagger \sin\theta$ is the analogous operator

Now consider the Hamiltonian

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{V}{2}(\hat{a}\hat{a} + \hat{a}^\dagger \hat{a}^\dagger) \tag{1}$$

(c) Diagonalize the Hamiltonian (1) using the $\hat{b}$ operators for suitably chosen η

(d) Show there is a limit on V for which this Hamiltonian makes physical sense