University of California at San Diego – Department of Physics – TA: Shauna Kravec

Quantum Mechanics A (Physics 212A) Fall 2016 Worksheet 3

Announcements

- The 212A web site is:

  http://keni.ucsd.edu/f16/ .

  Please check it regularly! It contains relevant course information!

Problems

1. **Quis Custodiet Ipsos Custodes?** (From Jacobs)

   Projective measurements lead to some weird things.

   Consider a two state system with basis vectors $\{|0\rangle, |1\rangle\}$. We are going to evolve the system according the Hamiltonian $\hat{H} = \frac{\omega}{2} Y$ where Y is the Pauli matrix $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$.

   (a) What is the unitary operator associated with time evolution? Given an initial prepared state of $|\psi_0\rangle = |0\rangle$. Write an expression for $|\psi(t)\rangle$.

   (b) What is the probability, as function of time, to measure $|0\rangle$?

   (c) Suppose we study the system over the time interval $[0, T]$ where $T \gg \delta t \equiv \frac{T}{N}$. We perform a measurement, in this basis, at every time $\frac{T}{N}, \frac{2T}{N}, \cdots$ where N is large. Assuming each measurement is independent from the other, what's the probability that the spin *never* flips to $|1\rangle$?

   (d) Evaluate this probability in the limit of $N \to \infty$.
   This is called the *quantum Zeno effect*.

2. **Building Bloch's Theorem**

   Consider a 1D Hamiltonian with a periodic potential $V(x) = V(x + na)$ for $n \in \mathbb{Z}$ and a the lattice spacing.

   (a) Define the operator T^n by $T^n|x\rangle = |x + na\rangle$. Show this is a symmetry.

   (b) Assuming H has no shared degeneracy with T, show that any eigenfunctions of this system can be chosen to obey

   $$\psi_k(x - a) = e^{-\mathbf{i}ka}\psi_k(x) \tag{1}$$

   Recall that $T|k\rangle = e^{-\mathbf{i}ka}|k\rangle$ and $\langle x|k\rangle \equiv \psi_k(x)$.

(c) Infer from (1) that one can then write $\psi_k(x) = e^{\mathbf{i}kx}u_k(x)$ where $u_k(x) = u_k(x+a)$

Note that k is different from our usual momentum. It's a *crystal momentum*!

(d) Show explicitly that for $P = -\mathbf{i}\partial_x$ that $P\psi_k(x) \neq k\psi_k(x)$

(e) Show that $\frac{-\pi}{a} \leq k \leq \frac{\pi}{a}$. What is $k + \frac{2\pi}{a}$?