10/5/16 Lecture 4 outline

• Last week: two state system, e.g. happy cat or sad cat. Can only measure one or the other, mutually exclusive, and a general state is a linear superposition with complex coefficients. Physical observables are Hermitian operators. Can work in basis of their eigenstates. We started to discuss $\vec{S} = \frac{1}{2}\hbar\vec{\sigma}$ in the $|\pm_{\hat{z}}\rangle$ basis. Note that $\vec{S}^2 = (3/4)\hbar^2\mathbf{1}$, and $[S^a, S^b] = i\hbar\epsilon^{abc}S^c$, which are basis independent. We wrote $|\pm_{\hat{x}}\rangle$ and $|\pm_{\hat{y}}\rangle$ in the $|\pm_{\hat{z}}\rangle$ basis.

• Stern Gerlach again, using bras and kets.

• Note that $|\pm_{\hat{x}}\rangle$ and $|\pm_{\hat{y}}\rangle$ are related to $|\pm_z\rangle$ by unitarity transformations. Aspects of unitary transformations and change of bases. Indeed, these transformations are examples of rotations. $U(\vec{\theta}) = e^{-i\vec{\theta}\cdot\vec{S}/\hbar}$, e.g. $U(2\pi) = -1$. Rotate $\pi/2$ around y axis to rotate z into x eigenstates. More generally, observables A and UAU^{-1} are unitary equivalent.

• It follows from $[S_a, S_b] \neq 0$ for $a \neq b$ that spin along different axes cannot be simultaneously diagnonalized, and hence they cannot be simultaneously measured.

• Schwarz inequality: $|||\chi\rangle||^2 = \langle\chi|\chi\rangle \geq 0$. Apply to $|\chi\rangle = |\alpha\rangle + x|\beta\rangle$ and minimize in x, taking it to be $-\langle\beta|\alpha\rangle/\langle\beta|\beta\rangle$, find $\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle \geq |\langle\alpha|\beta\rangle|^2$. Use this to prove $\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4}|\langle[A,B]\rangle|^2$ for Hermitian A and B. Use $\Delta A\Delta B = \frac{1}{2}[\Delta A, \Delta B] + \frac{1}{2}\{\Delta A, \Delta B\}$ and note that the first term is anti-Hermitian and the second is Hermitian, so their expectation values are pure imaginary and real, respectively.

• Compute dispersion of $S_{x,y,z}$ in $|+_{\hat{z}}\rangle$ state. Check above inequality for $A = S_a$ and $B = S_b$, so $[A, B] = i\hbar C$, with $C = S_c$ and $\epsilon_{abc} = 1$.