9/28/16 Lecture 2 outline

• Quantum mechanics: quantization of various classically continuous things, e.g. angular momentum, energy levels of atoms and other bounded systems (e.g. SHO). Recover approximate classical answers when quantum n is huge, e.g. when $S \gg \hbar$. We will start with a 2-state system because it is both mathematically simplest and physically as quantum as you can get.

• Stern-Gerlach [1921-1922] effectively measured the spin of an electron along an axis of choice. Send silver atoms through a gradient of $\vec{B}$, so $\vec{F} = \nabla(\vec{\mu} \cdot \vec{B})$, with $\vec{\mu} = ge\vec{S}/2mc$.

Instead of getting a continuous distribution, as expected classically, get two possibilities. Effectively measuring S_z and showed that the only possible outcomes are $S_z = \pm\frac{1}{2}\hbar$ (recall $\hbar c = 1973 eV\mathring{A}$). For an initially unpolarized, random sample, find half are spin up and half are spin down. The expected value $\langle S \rangle$ is thus zero, even though no experiment measures zero. The expected value of $\langle S_z^2 \rangle = (\hbar/2)^2$. Also $\langle (\Delta S_z)^2 \rangle = (\hbar/2)^2$, where $\Delta A \equiv A - \langle A \rangle$ is the deviation from average.

• Recall polarization sheets. Write $\vec{E}$ of an electromagnetic wave as $\vec{E} = Re\vec{E}_0 e^{i\vec{k}\cdot\vec{x}-\omega t}$. Here the i is just for convenience, of course $\vec{E}$ is real. Mentioning this because quantum states are naturally complex. As mentioned last time: $[x, p] = i\hbar$, emphasize the i. Take $\vec{k} = k\hat{z}$ and discuss choices of $\vec{E}_0$. Actually, $\vec{E}$ is a coherent collections of photons, and each has a spin variable – it is spin 1. The quantum description is similar to SG, but with complications because photons are bosons and they are massless – it really requires quantum field theory rather than non-relativistic QM. This class will only discuss non-relativistic QM.

• So we consider successive SG measurements along various axes and the analogy with light polarization. First consider blocking the $S_z = -\frac{1}{2}\hbar$ channel and then doing the experiment again, measuring along the z axis. As long as $\vec{B}$ is negligible in between (to avoid spin precession), find now that all the electrons are still spin up. If the first SG setup measures S_z and blocks down spins from passing, and the second measures S_z and blocks up spins from passing, nothing will pass through.

Now the fun: suppose that we put another SG setup in between. If that measures S_z, the results are unchanged. But if it measures instead $\vec{S}$ along another axis, now some do pass through all three. (It doesn't matter whether or not the middle one has one side blocked.) This shows that the middle measurement is not innocuous, but rather has changed the state of the system.

• Kets, operators, bras, basis kets, orthogonality and completeness relations.

• Physical observables correspond to Hermitian operators. Show that their eigenvalues are real and that bras and kets corresponding to different eigenvalues are orthogonal.

• $\vec{S} = \frac{1}{2}\hbar\vec{\sigma}$ in the $|\pm\widehat{z}\rangle$ basis. Note that $\vec{S}^2 = (3/4)\hbar^2\mathbf{1}$.

• Write $|\pm\widehat{x}\rangle$ and $|\pm\widehat{y}\rangle$ in the $|\pm\widehat{z}\rangle$ basis.

Ended here

• Stern Gerlach again, in this notation. Compute dispersion of $S_{x,y,z}$ in $|+\widehat{z}\rangle$.

• Use the Schwarz inequality $\langle\alpha|\alpha\rangle\langle\beta|\beta\rangle \geq |\langle\alpha|\beta\rangle|^2$ to prove $\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4}\langle[A,B\rangle|^2$ for Hermitian A and B. Use $\Delta A\Delta B = \frac{1}{2}[\Delta A, \Delta B] + \frac{1}{2}\{\Delta A, \Delta B\}$ and note that the first term is anti-Hermitian and the second is Hermitian, so their expectation values are pure imaginary and real, respectively.

• Position eigenstates. Momentum as generator of translations. Converting between position and momentum eigenstate bases. Translation generator $U(\vec{a}) = e^{-i\vec{p}\cdot\vec{a}/\hbar}$, satisfies $U(\vec{a})|\vec{x}\rangle = |\vec{x}+\vec{a}\rangle$, so $\langle\vec{x}|\psi\rangle = \psi(\vec{x})$ and $\langle\vec{x}|U|\psi\rangle = \psi(\vec{x}-\vec{a})$.