9/26/16 Lecture 1 outline

• Start with some big picture words. Details to follow.

• History: Old question - is light a bunch of particles, or a wave (in what)? Newton and others showed interference. Maxwell's equations showed it can be an electromagnetic wave, with speed c agreeing with observations of light's speed. (This eventually led to special relativity, but we won't discuss that here.) But there were plenty puzzles surrounding thermodynamics and observations of specific heats of non-ideal gasses and solids.

Around 1900, people realized that thermodynamics and electricity and magnetism don't fit together. Recall modes of solutions of the vacuum wave equations for electromagnetic waves: $Vd^3\vec{k}/(2\pi)^3$ times 2 polarizations, so number of modes between k and $k+dk$ is $2V(4\pi)k^2dk/(2\pi)^2$ and we can use $\omega = ck$. [Aside: waves on a string with fixed ends have $2L = n\lambda$ whereas with periodic boundary conditions, for both the wave and its derivative, the quantization is $L = n\lambda$, so $k = 2\pi n/L$; we can also see this by writing $e^{ikx}$and imposing $x \sim x + L$ periodicity, so $e^{ikL} = 1$. In 3d, get $\vec{k}_i = 2\pi\vec{n}_i/L_i$. For large V, we can replace the mode sum with an integral $\sum_{\vec{n}} \to \int d^3\vec{n}$ where $d^3\vec{n} = V(d^3\vec{k}/(2\pi)^3$.] Distribution $\sim \omega^2 d\omega$ and equipartition suggests each has energy kT, so get nonsensical UV catastrophy. Planck (1900) replaced $\omega^2 kT \to \hbar\omega^3(e^{\hbar\omega/kT} - 1)^{-1}$ by fitting data. New constant. Simple derivation from the stat mech of photons (identical Bosons) and $P(E) \sim e^{-E/kT}$ and Planck's formula is an example of the Bose-Einstein distribution; we won't discuss it, or T at all in this class.

1905: Einstein interpreted this, and the photo-electric effect as saying that light came in quanta – photons – with $E = \hbar\omega$. For many years, this was viewed as a nutty idea, and it indeed didn't fully make sense until other things were later understood about quantum mechanics (things that Einstein himself didn't believe).

1913: Bohr considered the puzzle that arose from Rutherford's observation that atoms are a bit like little solar systems, with the electron orbiting the heavy nucleus. Orbiting charge radiates and the atom would be classically unstable. Bohr proposed quantized energy levels, based on assuming $mvr = n\hbar$, and matched spectral lines to the transitions between these levels. It worked!

1916: Sommerfeld generalized the quantization to $\oint p_a dq_a = 2\pi n_a \hbar$.

1923: de Broglie: $\vec{p} = \hbar\vec{k}$. Matter waves. Leads to "particle or wave?" confusions.

1926: Schrodinger equation equation: $\widehat{H}\psi = E\psi$, where H is the Hamilton with the replacement $\vec{p} \to -i\hbar\nabla$. He thought ψ represented a smearing of the particles.

1925: Heisenberg matrix formulation. Not fully formed.

1925-1926: Max Born (with Jordan) and Dirac independently replace $\vec{x}$ and $\vec{p}$ with operators obeying $[x_j, p_k] = i\hbar\delta_{jk}$, acting on an abstract vector space.

1926: Born interprets ψ in terms of probability rather than smearing, with probability $\sim |\psi|^2$. Einstein in letter to Born (Dec 4, 1926): *Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not bring us any closer to the secret of the "old one". I, at any rate, am convinced that "He" does not throw dice.*

1927: Heisenberg uncertainty principle.

1926-1930: Dirac clarifies and reconciles all of the approaches. Classical observables like $\vec{x}$, $\vec{p}$, H are replaced with operators acting on a Hilbert space. Their Poisson brackets are replaced with commutation relations, $\{A, B\} \to [\widehat{A}, \widehat{B}]/i\hbar$. The state of a system is a vector in this Hilbert space. As in classical mechanics, the Hamiltonian generates time translations, momentum generates translations, angular momentum generates rotations. The state can be mixtures of classically distinct states, like the superposition of Schrodinger's cat being alive and dead.

• The details of measurement in quantum mechanics is subtle, and has led to a thicket of confusion in terms of interpretation. As an initial oversimplification, we pretend that the measurement apparatus is classical. We then pretend that its interaction with the quantum system causes the wavefunction to collapse, projecting it on to the eigenstate of the operator that was measured, corresponding to the observed eigenvalue. This projection is similar to that of polarizing sheets, as we'll discuss in the Stern-Gerlach example. The collapse of the wavefunction is certainly an oversimplification that is at best approximately true when the apparatus is approximately classical compared with the system. Everything should actually be described by quantum time evolution and the literal collapse does not make sense if one thinks about thought experiments. Becomes more relevant in recent years, as quantum systems become bigger, e.g. for quantum computing. "Quantum mechanics in your face." Many worlds interpretation? For simplicity, we mostly stick to the old-school oversimplifications.

• This class will be entirely based on non-relativistic quantum mechanics. There is a clash between relativity and quantum mechanics, and they are reconciled by quantum field theory. There, $\vec{x}$ is not an operator. Instead, the fields are operators acting on a Hilbert space. This structure also applies in non-relativistic contexts. It is like quantizing continuum mechanics instead of point particle mechanics. Upshot: cannot define $\widehat{x}$ operator and particle number is not conserved, e.g. an electron can emit a photon, which can itself yield an electron-positron pair. We won't discuss it at all in this class (though it is my research topic).

• Polarization and interference effects in light. Think about it in terms of individual light quanta – photons. Actually, electrons are similar in terms of quantum mechanics and simpler because they are massive (so we can restrict to non-relativistic simplifications), and charge conservation helps.

• Double slit interference pattern for light as a wave, and for individual photons, and for individual electrons. Closing one slit can lead to more photons or electrons at places where there would have been destructive interference.

• Emphasize linearity of adding states corresponding to possibilities. Get interference because probabilities come from squaring the additive thing, as with adding electric fields and squaring to get intensity.

• Path integral interpretation (Feynman): $\int [d\vec{x}(t)] e^{iS[\vec{x}(t)]/\hbar}$. We will briefly discuss it in this class. It pays off much more in quantum field theory. Instant payoff: see how to recover the classical principle of least action when $S \gg \hbar$, from the stationary phase approximation for an integral with widely oscillating integrand.

• Emphasize the fact that quantum phases come from adding complex numbers. Quantum states are complex valued. Get real answers for physical observables.

• Stern-Gerlach. Measure spin of atom (electron) by sending it through a magnetic field, with $F_z \approx \mu_z \partial_z B_z$ and $\mu_z = geS_z/2mc$. Find $S_z = \pm\frac{1}{2}\hbar$. Recall $\hbar c = 1973 eV\AA$.