11/9/16 Physics 212a Homework 4, due Wednesday Nov 23

1. Sakurai 2.28(c). This is a three part question involving an electron in a shell that notices a magnetic field in a region that the electron does not directly explore, but you don't need to do parts (a) and (b) in detail. Instead, please do the following parts

(a) Write down $\widehat{H}$ in position space, including the effect of $\vec{A} \neq 0$. Take $\vec{A} = A(\rho)_\phi \widehat{\phi}$ and find $A(\rho)$ from Stokes theorem (or look at the equation in the book). Use cylindrical coordinates. Note also that locally $\vec{A} = \nabla f(\vec{x})$, but that f is not globally well defined because it involves ϕ.

(b) Write down the position space differential equation for the energy eigenstates (i.e. the time-independent S.E.). Use separation of variables to write it as three ordinary differential equations in ρ, ϕ, and z. Which of these equations are affected by $B \neq 0$?

(c) Note that the z differential equation is the same as for a particle in a box. Write down its solutions, and note that this gives the ℓ term in the given $E_{\ell mn}$.

(d) Recall that $\psi \to e^{iqf/\hbar c}\psi$ under a gauge transformation. Use this with the f found in part (a) to relate the $B \neq 0$ S.E. equations to the $B = 0$ S.E. equations. In other words, make use of the fact that $\vec{A}$ is almost the same as zero, up to a gauge transformation – except that the gauge transformation is not globally well-defined. Show that, despite the fact that f is not globally well defined, the resulting ψ is well defined (single valued) provided that B satisfies the given quantization condition.

2. Sakurai 2.32. Partition function question and verify $E_0 = -\partial_\beta \ln Z|_{\beta\to\infty}$ for a particle in a 1d box.

3. Sakurai 2.33: $\langle \vec{p}', t' | \vec{p}, t\rangle$ for free particle.

4. Sakurai 2.34. SHO question.

5. Consider a particle of mass m that is on a circular hoop of radius R. Let $\phi \sim \phi + 2\pi$ be the angular generalized coordinate for the particle. The particle has charge q and there is a constant, uniform magnetic field perpendicular to the plane of the hoop. The particle is otherwise free.

(a) Write the Lagrangian $L(\phi, \dot{\phi})$ and find the conjugate momentum p_ϕ and Hamiltonian $H(\phi, p_\phi)$. To include the effect of $\vec{B}$ you'll need to find $\vec{A}_{\widehat{\phi}}$ and please express your answers in terms of $\Theta \equiv q\Phi_B = qB\pi R^2$.

(b) The quantum state $\phi(\phi) \equiv \langle \phi|\psi\rangle$ should be periodic in ϕ. Find the energy eigenstates and eigenvalues.

(c) Verify that the physics of the above answer is periodic under $\Theta \to \Theta + 2\pi$.

6. Sakurai 3.15 $\vec{J}$ angular momentum question.

7. Sakurai 3.16. Verify that $\vec{L} = \vec{x} \times \vec{p}$ commutes with $\vec{p}^2$ and $\vec{x}^2$.

8. Sakurai 3.17: $\psi(\vec{x}) = (x + y + 3z)f(r)$ question.

9. Sakurai 3.18. Find expectation values of $L_{x,y,z}$ and $L^2_{x,y,z}$ in the $|\ell, m\rangle$ state. Also, use $L_x = [L_y, L_z]/i\hbar$ and $L_y = [L_z, L_x]/i\hbar$ to argue that L_x and L_y must have zero expectation value in an L_z eigenstate.