212 Homework 3, due 10/28/16

1. Sakurai 2.20, verify that the given expressions for $J_\pm$ and J_z in terms of $a_\pm$ and $a_p^\dagger m$ satisfy the given relations, where $\vec{J}^2 \equiv J_z^2 + \frac{1}{2}(J_+J_- + J_-J_+)$ (the problem in the book forgot to define $\vec{J}^2$). This is a good warmup for when we discuss angular momentum, since these are the angular momentum commutation relations.

2. Sakurai 2.23. Particle in a box question.

3. Sakurai 2.24. Attractive delta function potential – a classic question.

4. Sakurai 2.25. Time evolution if the delta function potential is suddenly switched off.

5. Sakurai 2.37. Verifying some equations in the book related to when there is a magnetic field.

6. Sakurai 2.38, the correction to $\widehat{p}$ when in a magnetic field.

7. Sakurai 2.39, commutation relations in a uniform magnetic field.

8.

(a) Verify (using the Schrodinger equation) that the probability current is still conserved for a charged particle in a magnetic field if we modify $\vec{j}$ to $\vec{j} = -i(\hbar/2m)\psi^* \nabla\psi - \psi\nabla\psi^*) - (q/mc)\psi^*\psi\vec{A}$.

(b) Verify that $\vec{j}$ is invariant under a gauge transformation $\vec{A} \to \vec{A} + \nabla f$, $\psi \to e^{\pm iqf/\hbar c}$; check which sign works, and verify also the the Schrodinger equation is also invariant under that transformation.

9. Consider a 3d harmonic oscillator: $H = \vec{p}^2/2m + \frac{1}{2}m\omega^2\vec{x}^2$.

(a) What is the energy of the second excited state (i.e. the state that has more energy than both the groundstate and the first excited state)?

(b) What is the degeneracy of the second excited state?

(c) Suppose that $|\psi(t=0)\rangle = (|110\rangle + |012\rangle)/\sqrt{2}$. Find $\langle\vec{x}^2\rangle$ for all $t > 0$. Recall that $|n_x n_y n_z\rangle = |n_x\rangle \otimes |n_y\rangle \otimes |n_z\rangle$.