10/21 Lecture outline

$\star$ **Reading for today's lecture: Coleman lecture notes pages 103-120 (skip parts about counterterms for now).**

- Last time:

$$\langle f|(S-1)|i\rangle = \langle f|Te^{-i\int d^4x :\mathcal{H}_I:(x)}|i\rangle \equiv i\mathcal{A}_{fi}(2\pi)^4\delta^{(4)}(p_f - p_i).$$

The initial states have momenta $p_1 \dots p_n$ and the final states have momenta $q_1 \dots q_m$. Need to strip off the momentum conserving delta function to get the amplitude. We saw last time that $[A] = 4 - n_i - n_f$. Note the normal ordering in $:\mathcal{H}_I:(x)$: we want the full Hamiltonian to be normal ordered so e.g. $|0\rangle$ has zero energy. If we don't normal order here, we'll have issues with some loops for the interaction. Later (next quarter) we'll discuss counter terms, and essentially here the normal ordering is throwing away things that we'd anyway have to subtract off with counterterms.

Look at some examples, and connect with Feynman diagrams. As a first, simple example consider the toy model for nucleons and mesons:

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi^2 - \mu^2\phi^2) + (\partial\psi^\dagger\partial\psi - m^2\psi^\dagger\psi) - g\phi\psi\psi^\dagger.$$

So $\mathcal{H}_I = g\phi\psi^\dagger\psi$.

Use $\phi \sim a + a^\dagger$ for "mesons," $\psi \sim b + c^\dagger$, and $\psi^\dagger \sim b^\dagger + c$. We'll say that b annihilates a nucleon N and $c^\dagger$ creates an anti-nucleon $\bar{N}$. Conservation law, conserved charge $Q = N_b - N_c$.

We were considering $N + N \to N + N$, to $\mathcal{O}(g^2)$. The initial and final states are

$$|i\rangle = b^\dagger(p_1)b^\dagger(p_2)|0\rangle, \qquad \langle f| = \langle 0|b(p_1')b(p_2').$$

The term that contributes to scattering at $\mathcal{O}(g^2)$ is **(don't forget the time ordering!)**

$$T\frac{(-ig)^2}{2!}\int d^4x_1 d^4x_2 \phi(x_1)\psi^\dagger(x_1)\psi(x_1)\phi(x_2)\psi^\dagger(x_2)\psi(x_2).$$

The term that contributes to $S-1$ thus involves

$$\langle p_1'p_2'| :\psi^\dagger(x_1)\psi(x_1)\psi^\dagger(x_2)\psi(x_2): |p_1p_2\rangle = \langle p_1'p_2'| :\psi^\dagger(x_1)\psi^\dagger(x_2)|0\rangle\langle 0|\psi(x_1)\psi(x_2)|p_1, p_2\rangle.$$

$$= \left(e^{i(p_1'x_1 + p_2'x_2)} + e^{i(p_1'x_2 + p_2'x_1)}\right)\left(e^{-i(p_1x_1 + p_2x_2)} + e^{-i(p_1x_2 + p_2x_1)}\right).$$

The amplitude involves this times $D_F(x_1 - x_2)$ (from the contraction), with the prefactor and integrals as above. The final result is

$$i(-ig)^2 \left[\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_1 - p_2')^2 - \mu^2} \right] (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2').$$

Explicitly, in the CM frame, $p_1 = (\sqrt{p^2 + m^2}, p\widehat{e})$ and $p_2 = (\sqrt{p^2 + m^2}, -p\widehat{e})$, $p_1' = (\sqrt{p^2 + m^2}, p\widehat{e}')$, $p_2' = (\sqrt{p^2 + m^2}, -p\widehat{e}')$, where $\widehat{e} \cdot \widehat{e}' = \cos\theta$, and get

$$\mathcal{A} = g^2 \left(\frac{1}{2p^2(1 - \cos\theta) + \mu^2} + \frac{1}{2p^2(1 + \cos\theta) + \mu^2} \right).$$

According to the above, $[\mathcal{A}(2 \to 2)] = 0$ and the above is consistent with that. Good.

As we'll discuss, scattering by ϕ exchange leads to an attractive Yukawa potential. This was Yukawa's original goal, to explain the attraction between nucleons.