10/12 Lecture outline

⋆ **Reading for today's lecture: Coleman lecture notes, page 80-82.**

• Last time: recall that $[\phi(x), \phi(y)] = D_1(x-y) - D_1(y-x)$, where

$$\langle 0|\phi(x)\phi(y)|0\rangle = D_1(x-y) \equiv \int \frac{d^3k}{(2\pi)^3 2\omega(k)} e^{-ik(x-y)}.$$

We considered the Green's function for $\partial^2 + m^2$ and introduced the Feynman propagator: go above the $k_0 = E_k$ pole and below the $k_0 = -E_k$ pole

$$D_F = \theta(x_0 - y_0)D_1(x-y) + \theta(y_0 - x_0)D_1(y-x)$$

$$D_F(x-y) \equiv \langle T\phi(x)\phi(y)\rangle = \begin{cases} \langle\phi(x)\phi(y)\rangle & \text{if } x_0 > y_0 \\ \langle\phi(y)\phi(x)\rangle & \text{if } y_0 > x_0 \end{cases}.$$

$$D_F(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} e^{-ik(x-y)}.$$

Feynman's intuition: expected (retarded) propagation for normal matter ($k_0 = E_k$) and backwards-in-time propagation for antimatter. Will connect with Feynman diagrams, but we'll obtain it all from Dyson's derivation in terms of usual QM ideas. We'll see that time ordering arises this way.

• Explain the $i\epsilon$ and how the pole placement is such that the contour can be rotated to be along the imaginary k_0 axis, running from $-i\infty$ to $+i\infty$. This will later tie in with a useful way to treat QFT, by going to Euclidean space via imaginary time. It is something of a technical trick, but there is also something deep about it. Analyticity properties of amplitudes is deeply connected with causality. More later.

• Define contraction of two fields $A(x)$ and $B(y)$ by $T(A(x)B(y)) - :A(x)B(y):$. This is a number, not an operator. Let e.g. $\phi(x) = \phi^+(x) + \phi^-(x)$, where ϕ^+ is the term with annihilation operators and ϕ^- is the one with creation operators (using Heisenberg and Pauli's reversed-looking notation). Then for $x^0 > y^0$ the contraction is $[A^+, B^-]$, and for $y^0 > x^0$ it is $[B^+, A^-]$. So can put between vacuum states to get that the contraction is $\langle TA(x)B(y)\rangle$. For example, in the KG theory the contraction of $\phi(x)$ and $\phi(y)$ is $D_F(x-y)$.

• Wick's theorem (we'll soon see it's useful, since S-matrix elements will involve T ordered correlation functions):

$$\begin{aligned} T(\phi_1 \dots \phi_n) &=: \phi_1 \dots \phi_n : + \sum_{contractions} : \phi_1 \dots \phi_n : \\ &=: e^{\frac{1}{2}\sum_{i,j=1}^n C(\phi_i\phi_j)\frac{\partial}{\partial\phi_i}\frac{\partial}{\partial\phi_j}} \phi_1 \dots \phi_n \end{aligned}:$$

(where C is the contraction symbol) to get rid of the time ordered products.

Prove Wick's theorem by iteration: define the RHS as $W(\phi_1 \ldots \phi_n)$ and we assume $T(\phi_2 \ldots \phi_n) = W(\phi_2 \ldots \phi_n)$ and want to prove then that $T(\phi_1 \ldots \phi_n) = W(\phi_1 \ldots \phi_n)$. WLOG, take $t_1 > t_2 \ldots t_n$ so $T(\phi_1 \ldots \phi_n) = \phi_1 T(\phi_2 \ldots \phi_n) = \phi_1 W(\phi_2 \ldots \phi_n) = \phi_1^- W + W\phi_1^+ + [\phi_1^+, W]$. The first two terms are normal ordered and give all contractions not involving ϕ_1, while the last gives all normal ordered contractions involving ϕ_1.

So note that

$$\langle T(\phi_1 \ldots \phi_n) \rangle \begin{cases} 0 & \text{for } n \text{ odd} \\ \sum_{\text{fullycontracted}} & \text{for } n \text{ even.} \end{cases}$$