215a Homework exercises 2, Fall 2015, due Oct. 14

"Tong problem $n.m$" refers to exercise set n, problem m. Follow links from website.

1. Consider a complex scalar field with

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi$$

Define

$$\phi(x) \equiv \int \frac{d^3k}{(2\pi)^3 2\omega_k} \left(a(k) e^{-ik\cdot x} + b^\dagger(k) e^{ik\cdot x} \right).$$

$$\phi^\dagger(x) \equiv \int \frac{d^3k}{(2\pi)^3 2\omega_k} \left(a(k)^\dagger e^{ik\cdot x} + b(k) e^{-ik\cdot x} \right).$$

(a) Find the conjugate coordinate $\Pi(x)$ to $\phi(x)$.

(b) Impose the canonical equal-time commutation relation $[\phi(\vec{x},t), \Pi(\vec{y},t)] = i\delta^3(\vec{x}-\vec{y})$ and show this implies that $[a(k), a^\dagger(k')] = [b(k), b^\dagger(k)] = (2\pi)^3 2\omega_k \delta^3(\vec{k}-\vec{k}')$, with all other commutators vanishing.

(c) Recall from HW1 that there is a conserved current, $j^\mu(x)$ with $\partial_\mu j^\mu = 0$, corresponding to the $\phi \to e^{i\alpha}\phi$ symmetry. Write the corresponding charge $Q = \int d^3x j^0$ as $Q = \int d^3k \ldots$, where $\ldots$ is in terms of things like $a(k)$ and $b(k)$. Write Q as a normal ordered expression, so $Q|0\rangle = 0$.

(d) Verify that $a^\dagger(k)|0\rangle$ and $b^\dagger(k)|0\rangle$ are eigenstates of Q. What are their eigenvalues?

2. Consider the KG theory $\mathcal{L}_{KG} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$.

(a) Let $|p\rangle$ be the one-particle state $a^\dagger(p)|0\rangle$. Show that

$$\langle 0|\phi(x)|p\rangle = e^{-ip\cdot x}.$$

(b) Using the expressions given in lecture for H and $\vec{P}$, show that

$$[P^\mu, \phi(x)] = -i\partial^\mu \phi(x).$$

3. Tong problem set 2, exercise 7 (non-relativistic theory).

4. Tong problem set 2, exercise 9 (Wick's theorem check).