Waves I

Lots of examples: light, sound, electrical, bio, quantum ...

Classical mechanics: mass on spring, pendulum, U-tube, .. molecules ...

Whenever small deviations away from equilibrium (& friction ignored) ~ mass on spring.

$$F = ma \longrightarrow m\frac{d^2x}{dt^2} = -kx$$

$$x(t) = A\cos(\omega t + \varphi_0)$$

Traveling waves e.g. light, sound, string

$$y = A\cos(kx - \omega t + \varphi_0)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T} \qquad v = \frac{\omega}{k} = \frac{\lambda}{T}$$

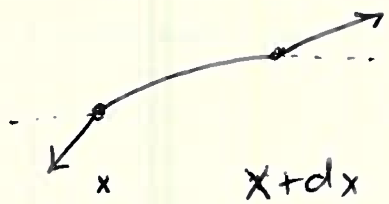
Sol'n of wave eqn:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

Elaborate on k, ω, nodes etc.

Derive for stretched string

$v = \sqrt{\tau/\mu}$ check dimensions.



$$\mu\, dx \, \frac{\partial^2 y}{\partial t^2} = \tau\left(\frac{\partial y}{\partial x}(x+dx) - \frac{\partial y}{\partial x}(x)\right) \Longrightarrow \frac{\partial^2 y}{\partial t^2}\left(\frac{\mu}{\tau}\right) = \frac{\partial^2 y}{\partial x^2}$$

$$v^2 = \tau/\mu$$

$$y(x,t) = A\cos(kx - \omega t + \varphi_0)$$

for any k with $\omega = vk$

$y(x + m\lambda,\ t + nT) = y(x,t)$ integer n, m

$$\omega = \frac{2\pi}{T} \qquad k = \frac{2\pi}{\lambda}$$

phase velocity v vs $\frac{\partial y}{\partial t} \equiv$ "transverse velocity"

Energy density

$$k(x,t) = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 \qquad u(x,t) = \frac{1}{2}\tau\left(\frac{\partial y}{\partial x}\right)^2$$

Energy transmission:

$$dK = \frac{1}{2}\, dm \left(\frac{\partial y}{\partial t}\right)^2$$

$$\frac{dK}{dt} = \frac{dx}{dt}\,\frac{dK}{dx} = v\; k(x,t)$$

Average power transmitted: $\langle P \rangle = 2\left\langle \frac{dK}{dt} \right\rangle$

$$= \frac{1}{2}\mu v \omega^2 A^2$$

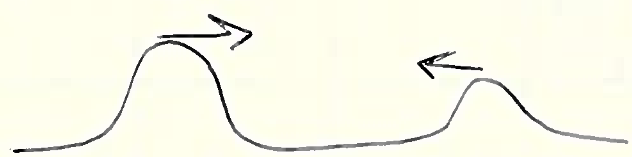
Note any $y(x,t) = f(kx - \omega t)$

satisfies wave eqn. More general sol'n

= superposition of waves.

e.g. $y_T(x,t) = y_1(x,t) + y_2(x,t)$

just add.



e.g. $$f(x,t) = \sum_i a_i \cos\left(k_i(x - ct) + \varphi_0\right)$$

or $$\int_{-\infty}^{\infty} dk\; a(k)\cos\left(k(x - ct) + \varphi_0\right)$$

$$= f(x - ct)$$

Interference: $y_1 = A \sin(kx - \omega t)$

$$y_2 = B \sin(kx - \omega t + \phi)$$

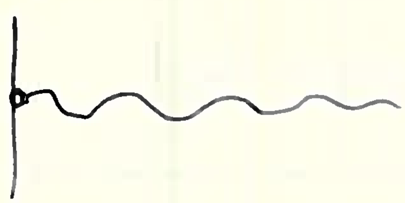
$$y_T = C \sin(kx - \omega t + \gamma)$$

e.g. for $A = B$ $\quad C = 2A \cos\frac{\phi}{2}, \; \gamma = \frac{\phi}{2}$

$\phi = \pi \rightarrow$ waves cancel

phasors $\rightarrow$ complex #s

Standing waves: waves traveling in opposite directions



tied to wall at $x = 0$

$$y = A_1 \sin(kx - \omega t) + A_1 \sin(kx + \omega t)$$

$$= 2A_1 \sin(kx) \cos(\omega t)$$

$$= A \sin(kx) \cos(\omega t)$$

Both ends tied to wall at $x = L$

$$y(L,t) = 0$$

$$\Rightarrow \quad A \sin(kL) \cos \omega t = 0$$

$$kL = n\pi = \frac{2\pi L}{\lambda}$$

or $\lambda_n = 2L/n$

etc..

n^{th} harmonic

Using $\omega = vk$ with $v = \sqrt{\tau/\mu}$

find $\omega_n = \sqrt{\frac{\tau}{\mu}} \frac{n\pi}{L} = n\omega_1$

↑ quantized