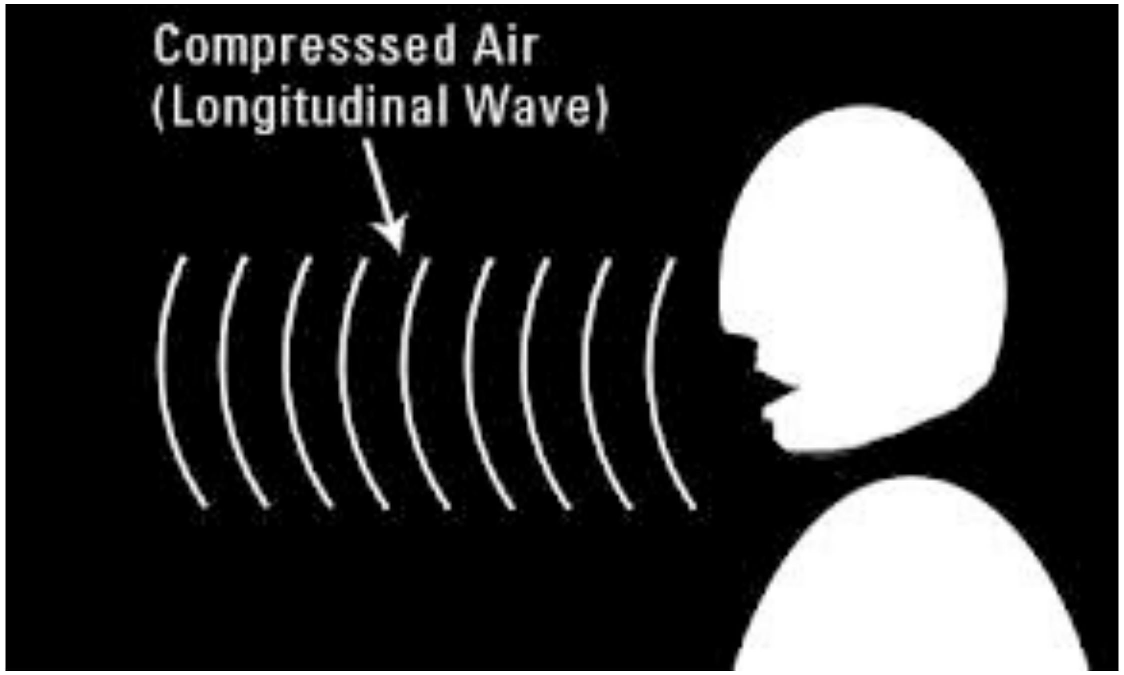
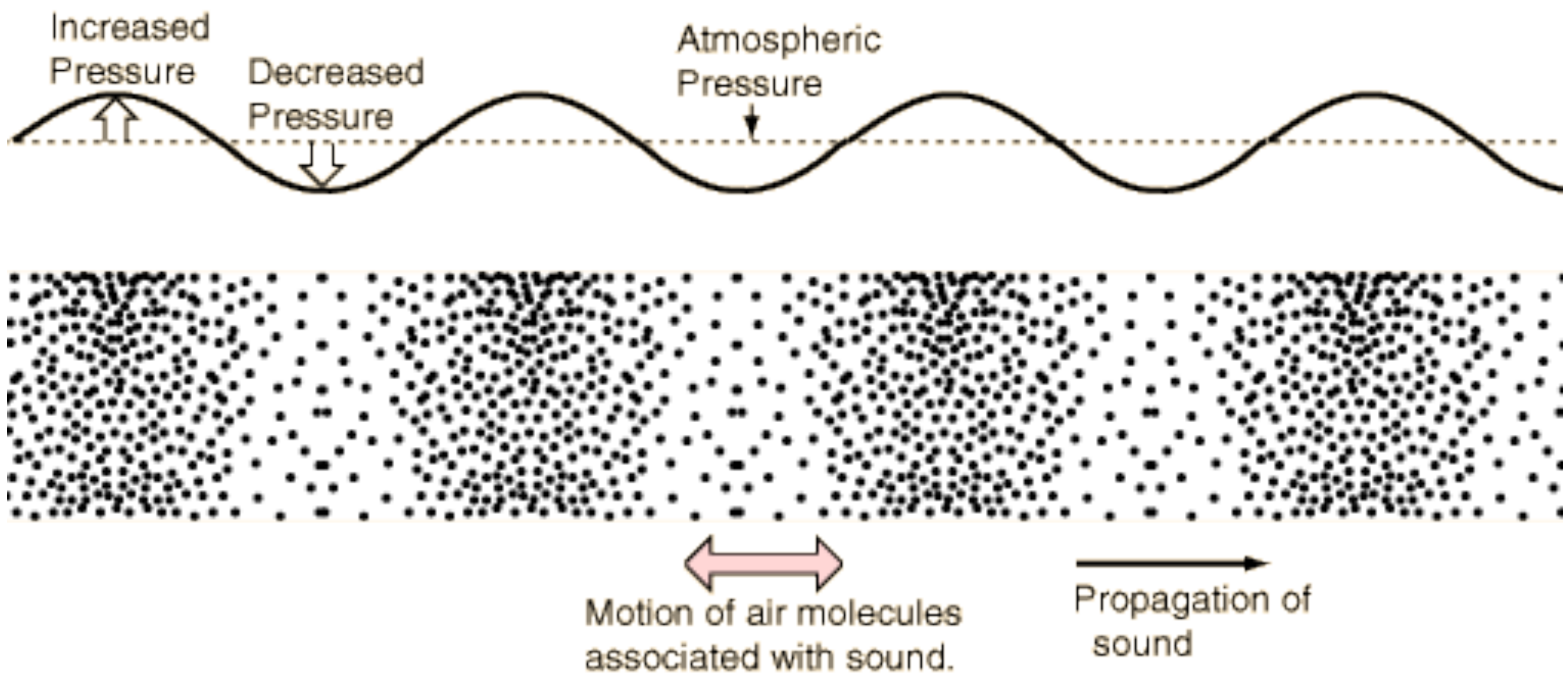
Sound Waves

Ken Intriligator's week 3 lectures, Oct 14, 2013



Ken: "OK, let's get started.."

Sound = pressure wave



$s(\vec{x}, t)$
displacement fluctuation

$s(\vec{x}, t)$ satisfies 3d wave equation: $(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2})s(\vec{x}, t) = 0$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 f(r) = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r f(r))$$

r = spherical, radial coordinate

3d wave eqn. solutions

"Plane wave" $s(\vec{x},t) = A\cos(\vec{k}\cdot\vec{x} - \omega t + \phi)$

$\vec{k} = k\widehat{n}$ $k = \dfrac{2\pi}{\lambda}$ $\widehat{n} =$ unit vector point in dir. of wave velocity

$\omega = vk$

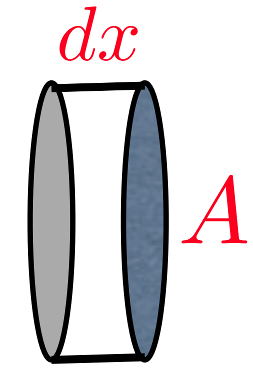
"Spherical wave" $s(r,t) = \dfrac{A}{r}\cos(kr - \omega t + \phi)$

$\omega = vk$

Consider first plane waves, in the x direction:

$$s(x,t) = A\cos(kx - \omega t + \phi)$$

Derive the wave eqn.:



$p(x,t)$ = gauge pressure $\quad p_{tot} = p_0 + p$

$p = -B(dV/V)$ B= "bulk modulus," it's just like the spring constant k in F = -kx.

("Incompressible" fluid has B=infinity)

$$p = -B\frac{\partial s}{\partial x}$$

$$F/V = (m/V)a$$

$$-\frac{\partial p}{\partial x} = B\frac{\partial^2 s}{\partial x^2} = \rho_0\frac{\partial^2 s}{\partial t^2}$$

$$\frac{\partial^2 s}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$$

$$v_{sound} = \sqrt{\frac{B}{\rho_0}}$$

Fits with D.A.!

Examples

$$v_{sound} = \sqrt{\frac{B}{\rho_0}}$$

Water: $v_{sound} \approx 1480 m/s$

$B = 2.18 \times 10^9 Pa$
$\rho_0 = 10^3 kg/m^3$

Air: $v_{sound} \approx 344 m/s$

$$v_{gas} = \sqrt{\frac{\gamma k T}{m}}$$

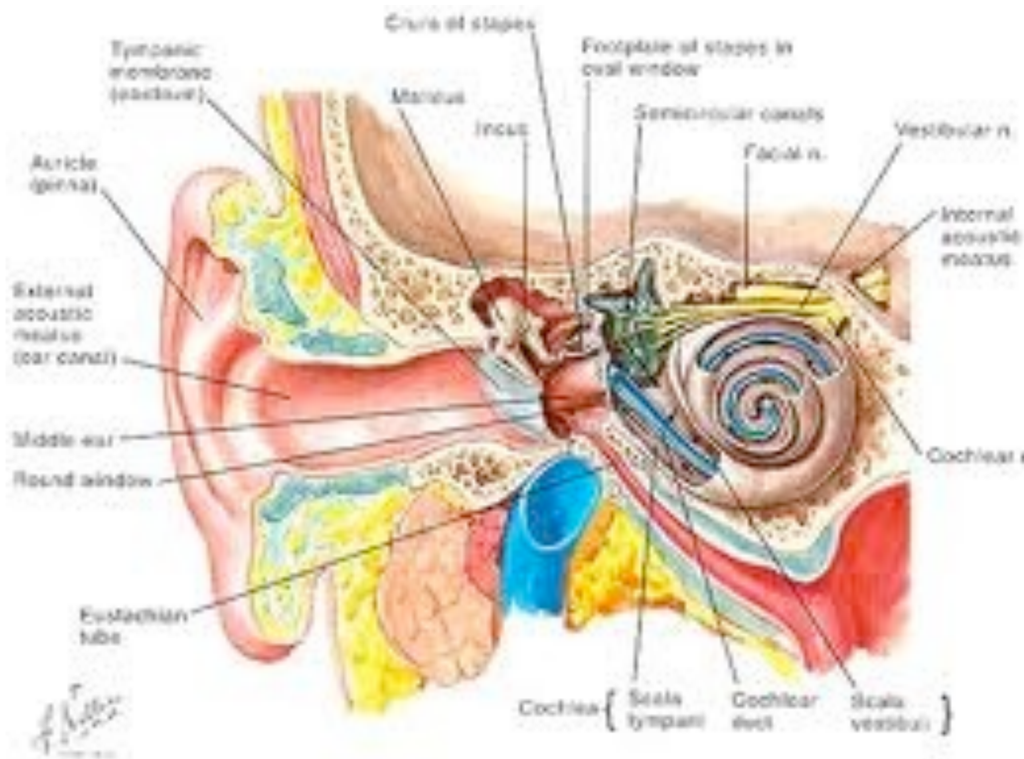
Ignore this now, for later.

We'll understand gasses later. For this week, ignore all parts of the chapter mentioning temperature T.

Ear's f (freq) ranges

$$v_{sound} = \lambda / T = \lambda f$$

$$v_{sound} \approx 344 m/s$$



Humans: $20Hz \leq f \leq 20,000Hz$

$1.7cm \leq \lambda \leq 17.5m$

Dogs: $40Hz \leq f \leq 60,000Hz$

Cats: $60Hz \leq f \leq 80,000Hz$

Bats: $10,000Hz \leq f \leq 200,000Hz$

Dolphin: $75Hz \leq f \leq 150,000Hz$

Piano

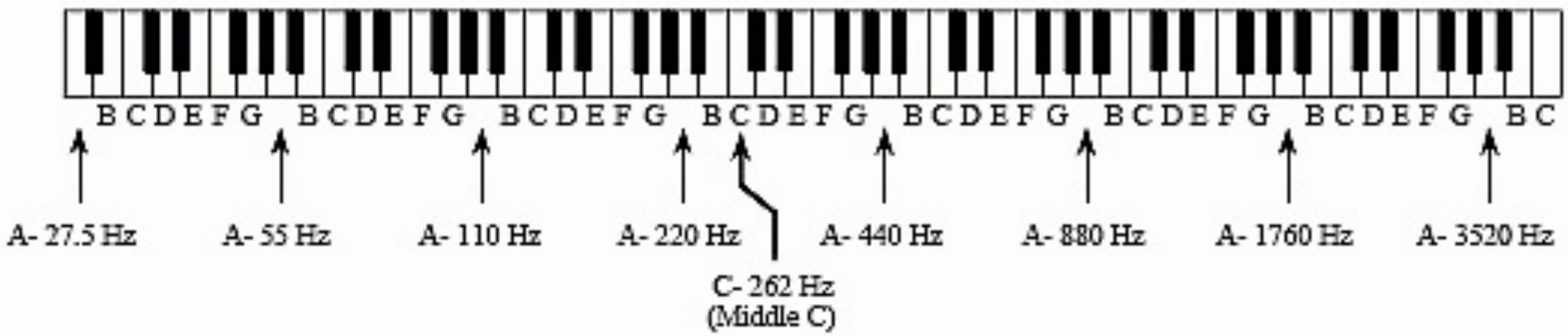


FIGURE 22-4
The Piano keyboard. The keyboard of the piano is a *logarithmic* frequency scale, with the fundamental frequency doubling after every seven white keys. These white keys are the notes: *A, B, C, D, E, F and G*.

Pressure Wave eqn.

Displacement wave equation:

$$\frac{\partial^2 s}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 s}{\partial t^2}$$

(Derived a few slides ago.)

Multiply both sides of the s wave eqn. by $\cdot B \frac{\partial}{\partial x}$

$$\downarrow$$

$$p = -B\frac{\partial s}{\partial x}$$

So get same wave eqn for pressure

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 p}{\partial t^2}$$

$$s(x,t) = A\cos(k(x-vt)+\phi_0)$$

$$p(x,t) = BkA\sin(k(x-vt)+\phi_0)$$

$$p_{max} = BkA = (v\rho)(A\omega)$$

Larger A (louder) or higher frequency gives bigger p, this can hurt your ears!

Ouch!

$$s(x,t) = A\cos(k(x - vt) + \phi_0)$$

$$p(x,t) = BkA\sin(k(x - vt) + \phi_0)$$

$$p_{max} = BkA = (v\rho)(A\omega)$$

Larger A (louder) or higher frequency gives bigger p, this can hurt your ears!

Largest gauge pressure before damaging your ears:

$p_{max} \approx 28Pa$ $\quad v \approx 343m/s$ $\quad \rho_{air} \approx 1.21kg/m^3$

$$(A\omega)_{max} \approx 28/(343)(1.21) \approx 0.067m/s$$

e.g. $f = 10^3 Hz = \omega/2\pi$ $\longrightarrow$ $A_{max} \approx 1.1 \times 10^{-5} m$

Sound wave amplitude, at this frequency, should be less, or you'll damage your ears.

Intensity

$$I = \frac{\overline{\text{Power}}}{\text{Area}}$$

Line on top means "average", sometimes we also use these brackets < > to denote averaging.

(Area). $I = \frac{\overline{dE}}{dt} = \frac{\overline{dK}}{dt} + \frac{\overline{dU}}{dt} = 2\frac{\overline{dK}}{dt}$

Like the harmonic oscillator, all oscillating systems have average K and U are equal.

$$\frac{dK}{dt} = \frac{1}{2}\frac{dm}{dt}\left(\frac{\partial s}{\partial t}\right)^2 = \frac{1}{2}\rho \text{Area}\frac{dx}{dt}\left(\frac{\partial s}{\partial t}\right)^2 \longleftarrow \quad s = s_{max}\cos(kx - \omega t + \phi)$$

$$= \frac{1}{2}\rho(\text{Area})v_{sound}\omega^2 s_{max}^2 \sin^2(kx - \omega t + \phi) \qquad (\overline{\sin^2(*)} = \frac{1}{2})$$

$$\longrightarrow \quad I = \frac{1}{2}\rho v_{sound}\omega^2 s_{max}^2$$

(For point source, s ~ 1/r)

$$I_{point\ source} = P_s/4\pi r^2$$

Intensity & sound level

$$\beta \equiv (10dB)\log_{10}(I/10^{-12}W/m^2)$$

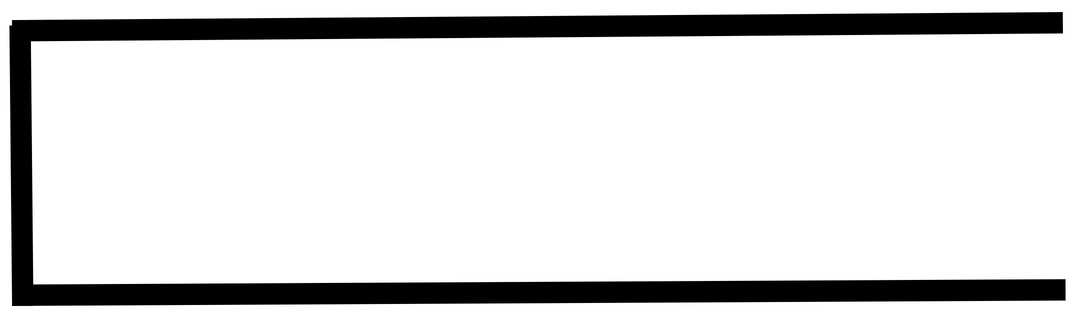
$I = 10^{-12}W/m^2 \rightarrow \beta = 0$ "What?" Too quiet to hear.

Human senses (hearing, seeing, touch, smell, taste) detect intensities I on a log scale. Which is good! Can detect over a large I range, from faint to huge.

Conversation has beta about 60dB. Loud rock concert is about 110dB. Pain threshold is about 120dB.

Waves in pipes!

s=0 at
closed ends



p (gauge) =0
at open ends

$$p = -B\frac{\partial s}{\partial x}$$

So, the wave eqn. BCs are:

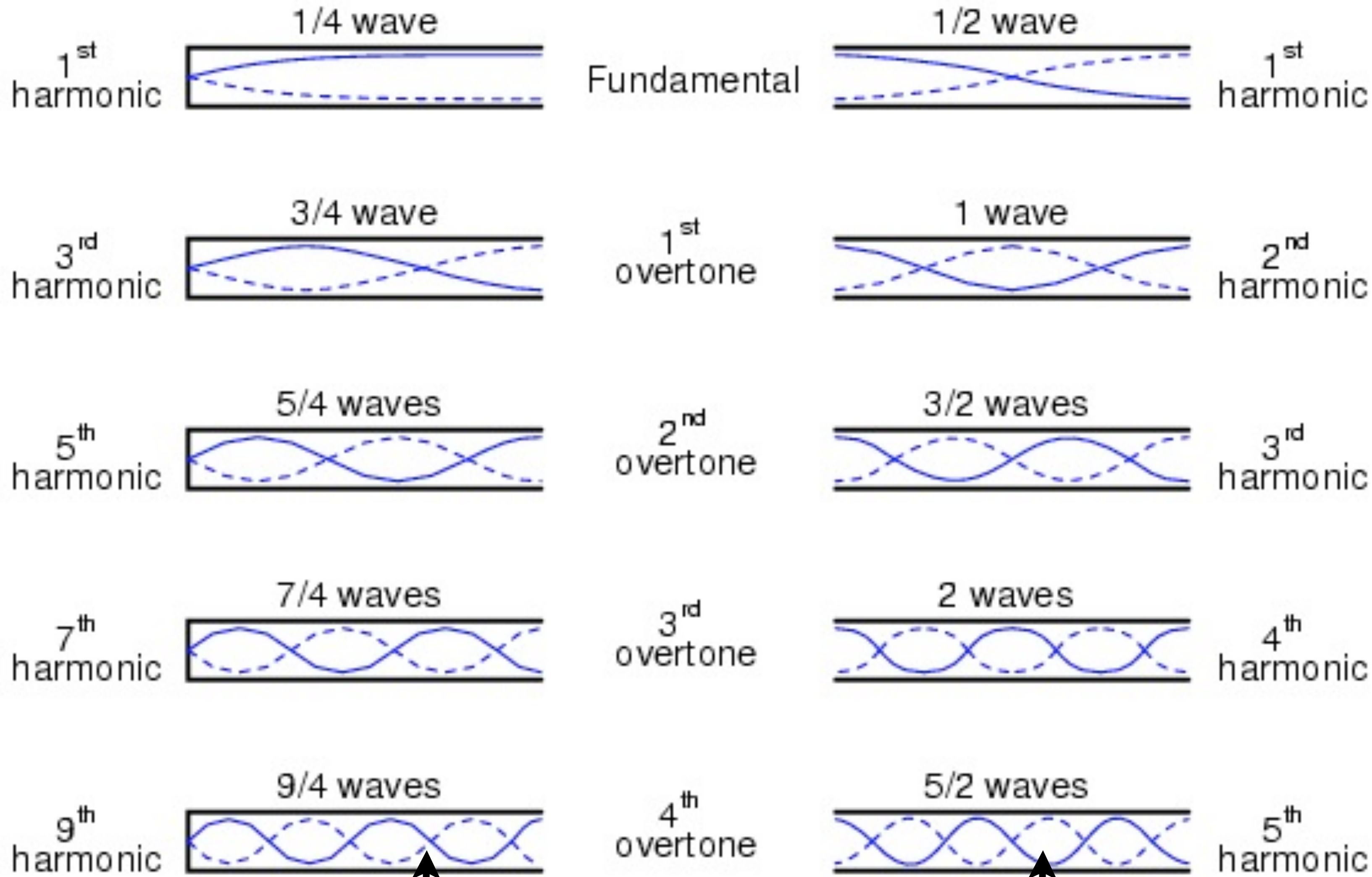
$$s|_{closed} = 0$$ and $$\frac{\partial s}{\partial x}|_{open} = 0$$

(Just like with a string at fixed end.)

(Just like with a string at a free end.)

Pipe wave harmonics

(Drawing wave's s displacement. Pressure is ~ derivative, so opposite.)



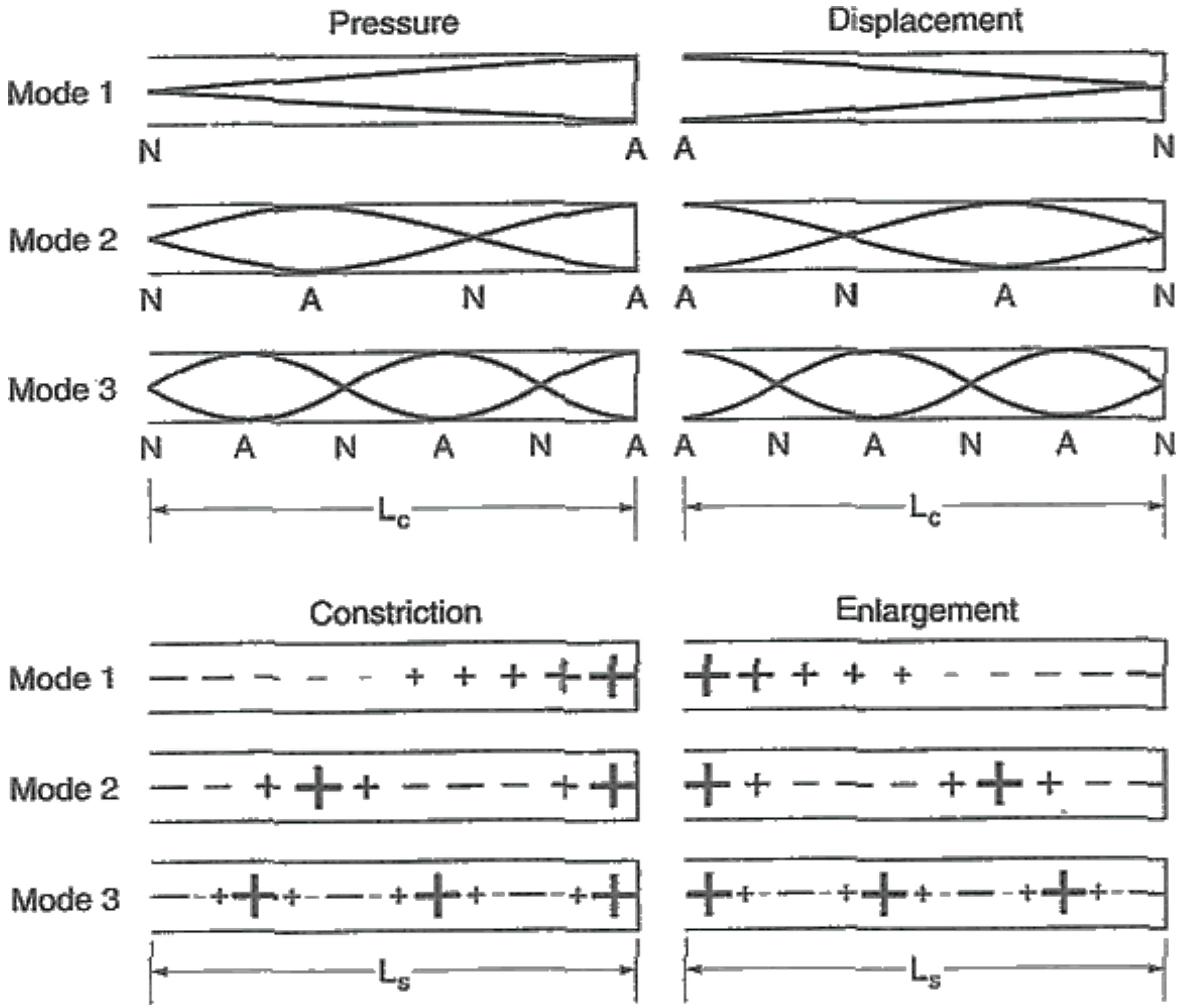
$$\lambda = 4L/n_{odd}$$

One end open, one end closed

$$\lambda = 2L/n$$

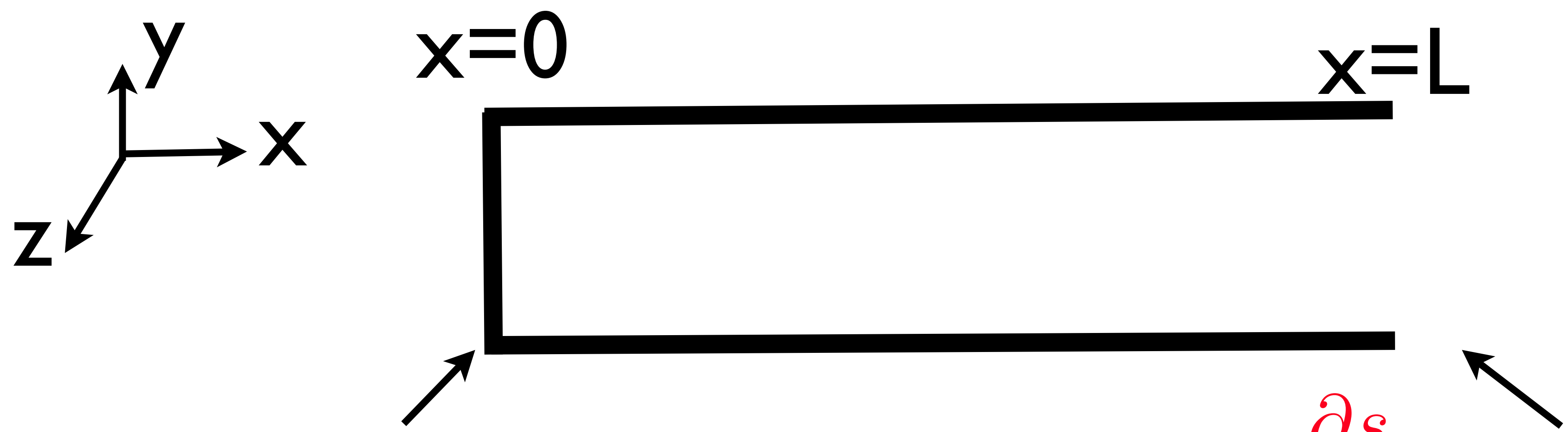
Both open, or both closed

s & p in pipes



Pressure
Displacement
Mode 1
N A A N
Mode 2
N A N A A N A N
Mode 3
N A N A N A A N A N A N
L_c
L_c
Constriction
Enlargement
Mode 1
Mode 2
Mode 3
L_s
L_s

The math for pipe wave



$$s(t, x=0) = 0$$

$$\frac{\partial s}{\partial x}(t, x=L) = 0$$

$$s = s_{max} \sin(kx) \cos(kvt + \phi_0)$$

B.C.@L: $\cos(kL) = 0 \longrightarrow kL = (n + \frac{1}{2})\pi$

$$k = 2\pi/\lambda$$

$$L = (2n+1)\lambda/4$$

Math for other cases

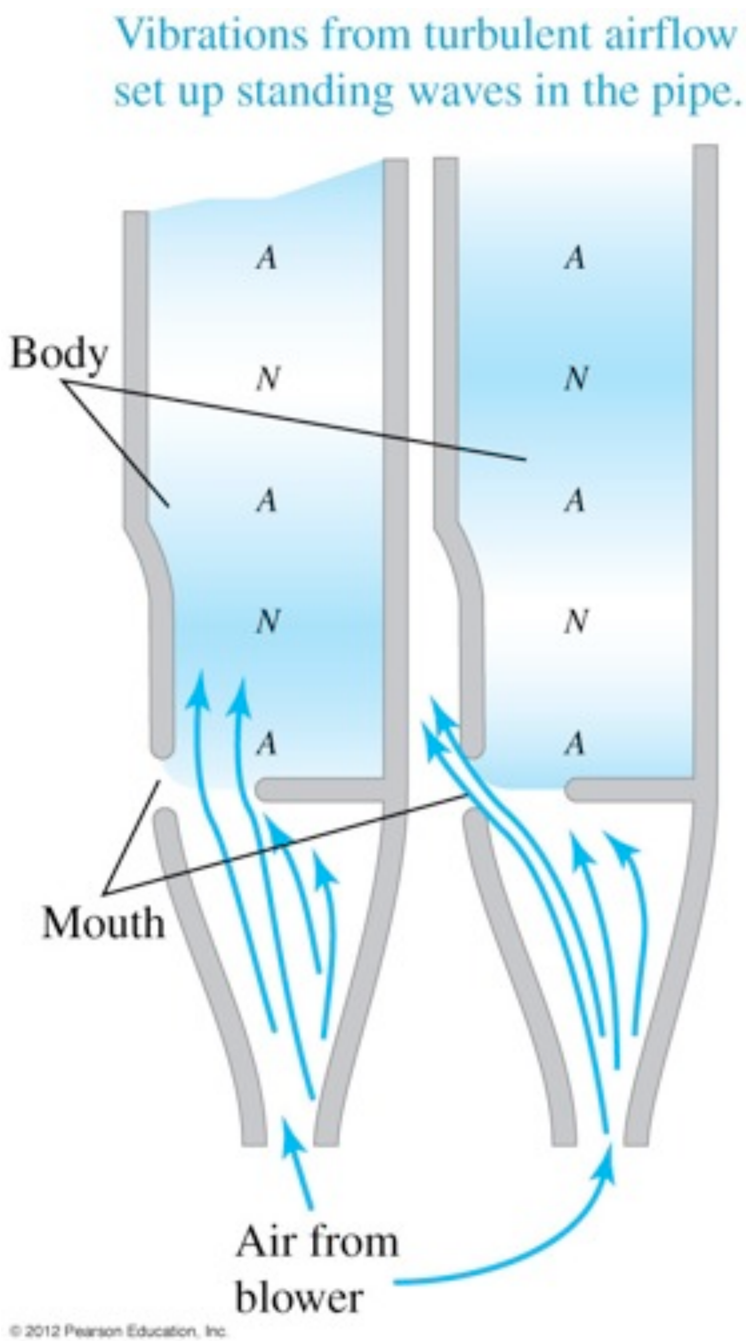
Both ends closed: $s = s_{max} \sin(kx) \cos(kvt + \phi)$

B.C.@L: $kL = n\pi \longrightarrow \lambda = 2L/n$

Both ends open: $s = s_{max} \cos(kx) \cos(kvt + \phi)$

B.C.@L: $kL = n\pi \longrightarrow \lambda = 2L/n$

FAQ: Why the sound?



From last time, get:

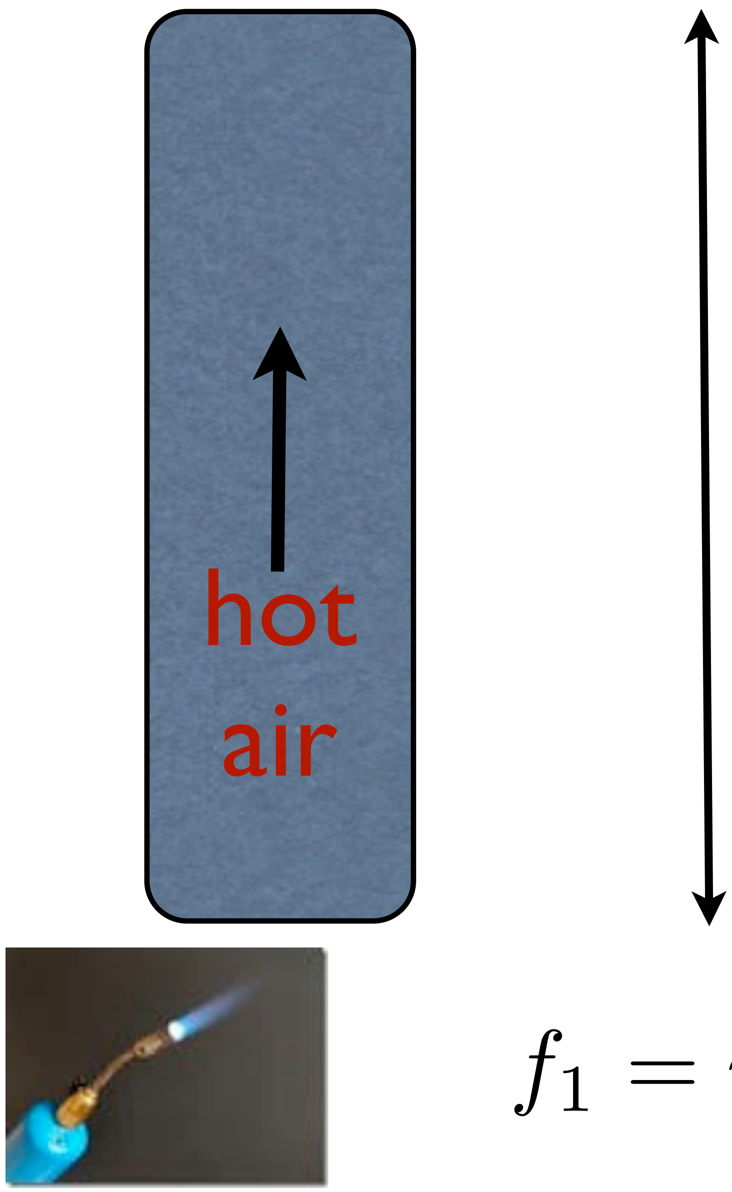
$$f_n = nf_1 = nv_{sound}/\lambda_1$$

OO or CC: $\lambda_1 = 2L, \ n = 1, 2, 3, 4 \ldots$

OC: $\lambda_1 = 4L, \ n = 1, 3, 5, 7 \ldots$

here odd only

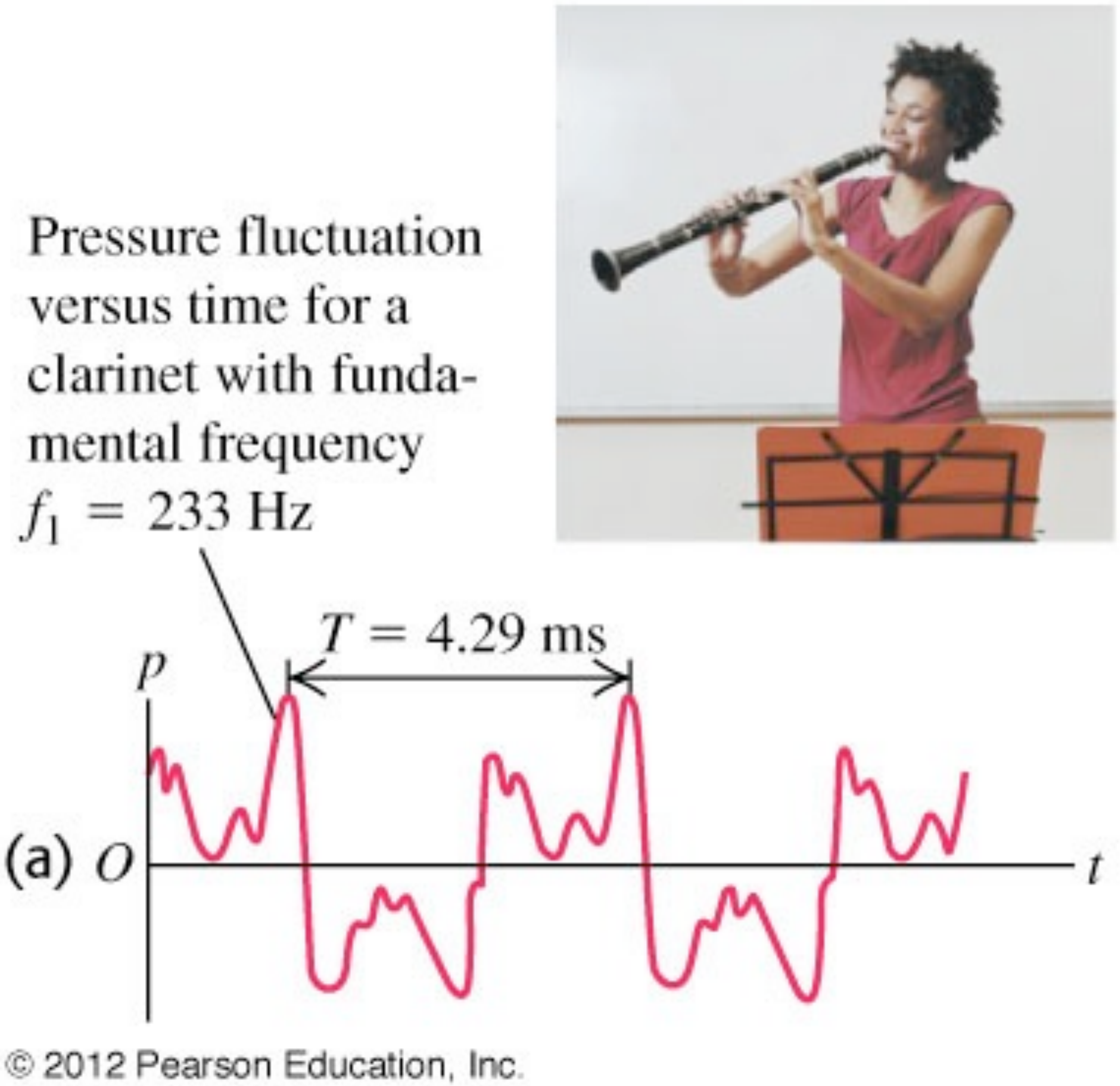
Example: singing tube



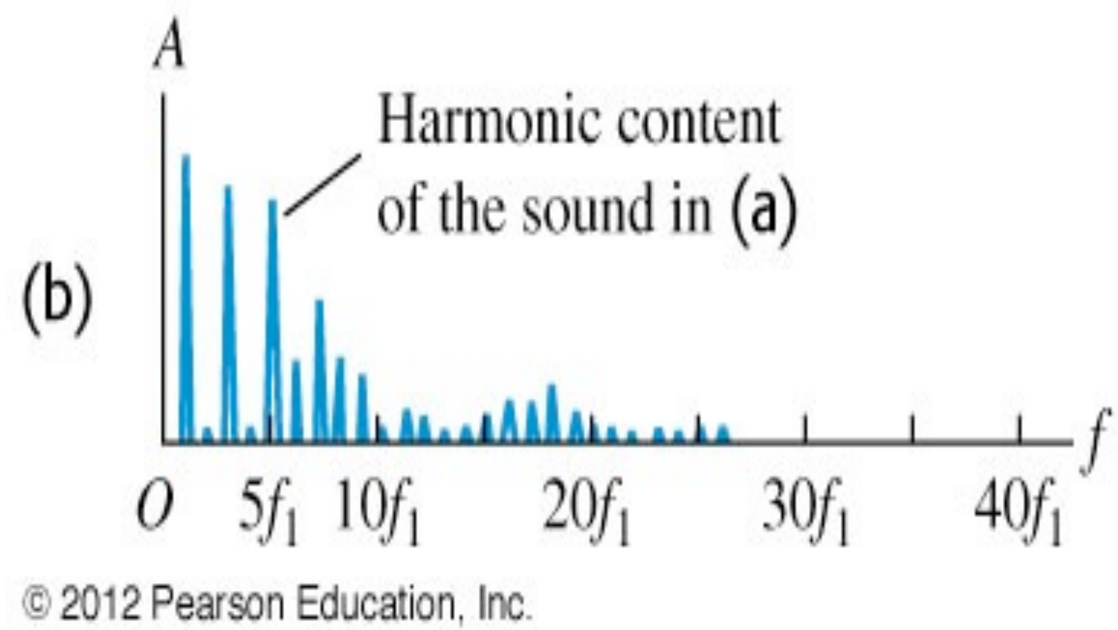
L (e.g. = 1m)

$$f_1 = v/\lambda_1 = v/2L = 344ms^{-1}/2m = 172Hz.$$

How does it sound?

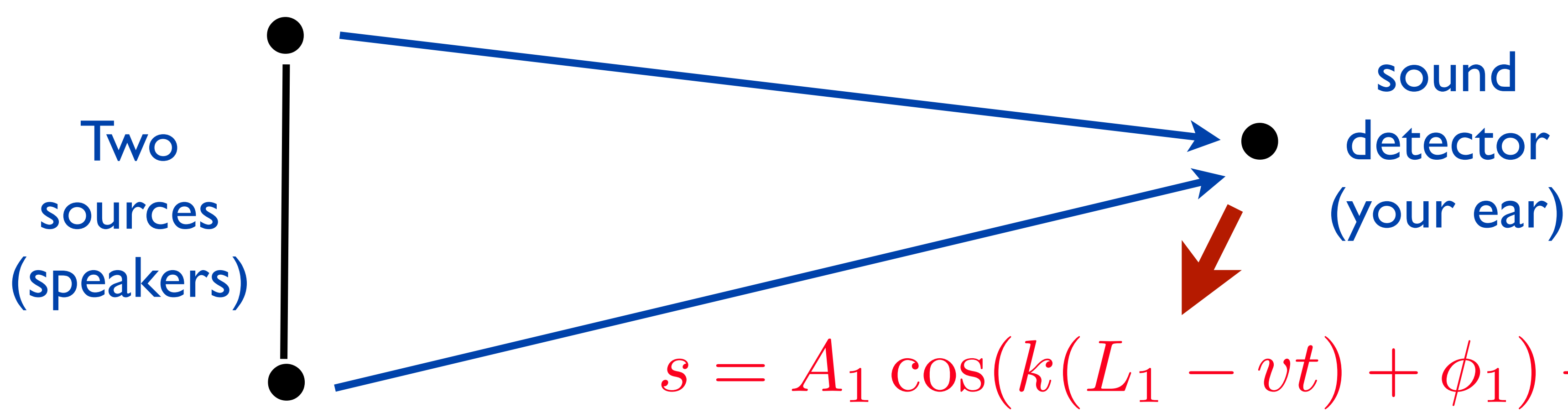


$$233Hz = v_{sound}/4L$$



Interference

Superpose two traveling waves.



Can be louder or quieter(!) than with just one speaker, depending on your location. Here's why:

$$s = A_1 \cos(k(L_1 - vt) + \phi_1) + A_2 \cos(k(L_2 - vt) + \phi_2)$$

Let's keep it simple, take: $A_1 = A_2$ $\quad \phi_1 = \phi_2$

use $$\cos a + \cos b = 2\cos(\frac{a-b}{2})\cos(\frac{a+b}{2})$$

get $$s = 2A\cos k(\frac{L_2 - L_1}{2})\cos(k(\frac{L_1 + L_2}{2}) - \omega t + \phi)$$

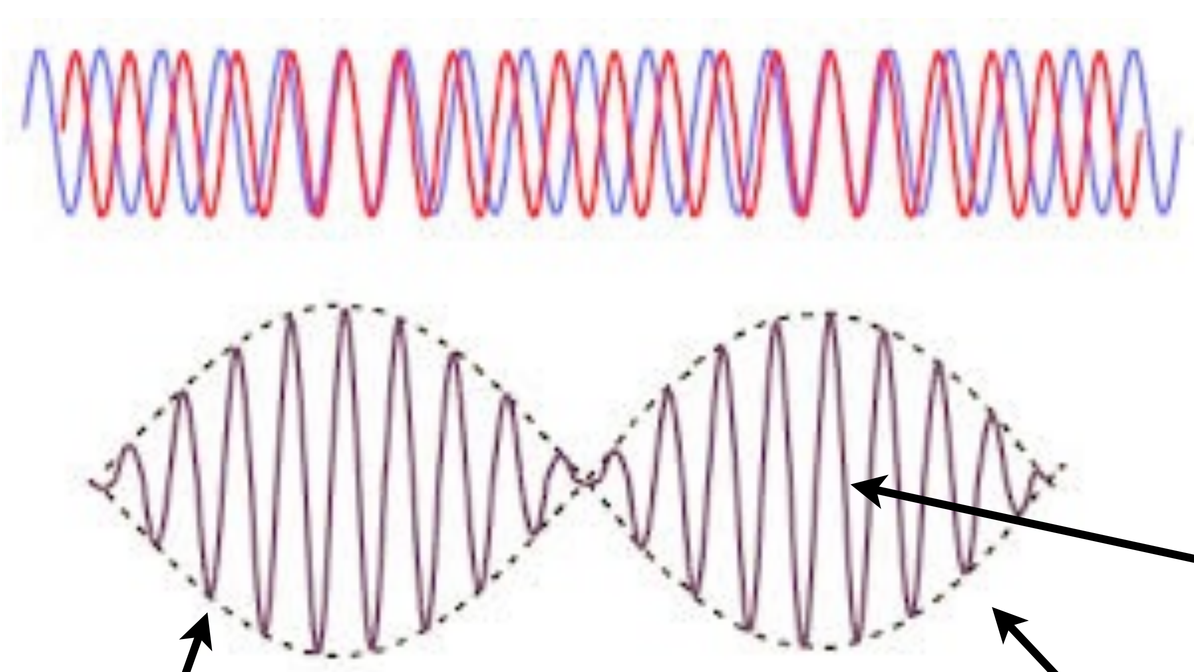
Constructive: $k\Delta L = 2\pi n$ i.e. $\Delta L = n\lambda$

Destructive: $k\Delta L = (2n+1)\pi$ i.e. $\Delta L = (n + \frac{1}{2})\lambda$

Beats

Superpose two sources with different frequencies:

$$A\cos(\omega_1 t) + A\cos(\omega_2 t) = 2A\cos(\frac{1}{2}(\omega_1 - \omega_2)t)\cos(\frac{1}{2}(\omega_1 + \omega_2)t)$$



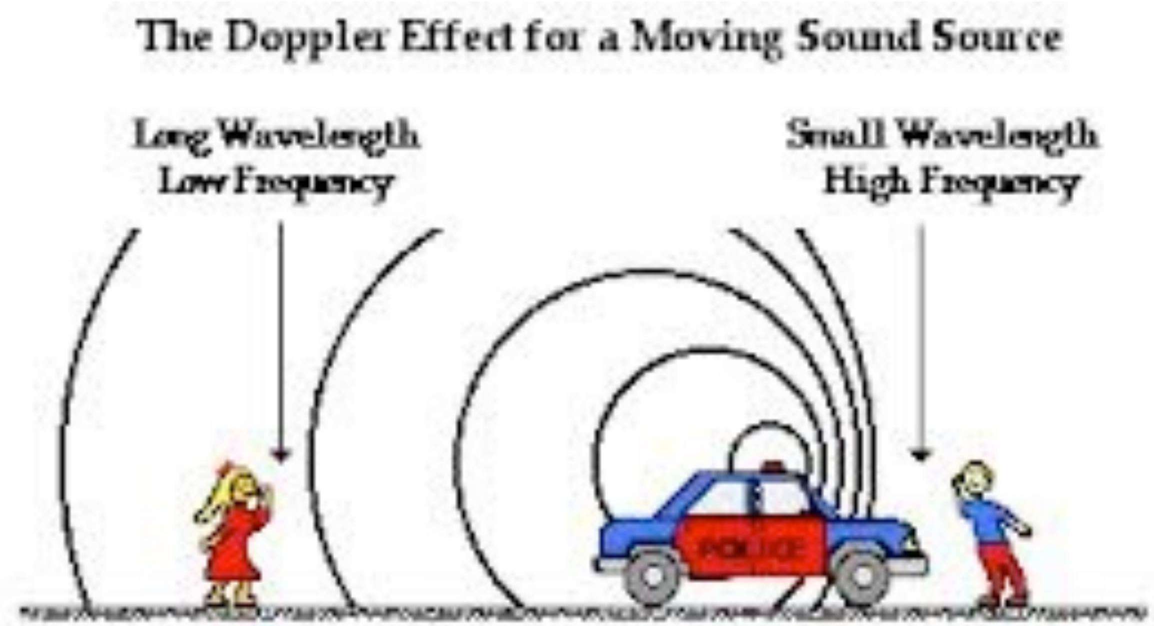
Over time, sometimes add constructively, sometimes destructively. Hear beats.

Inside, average frequency.

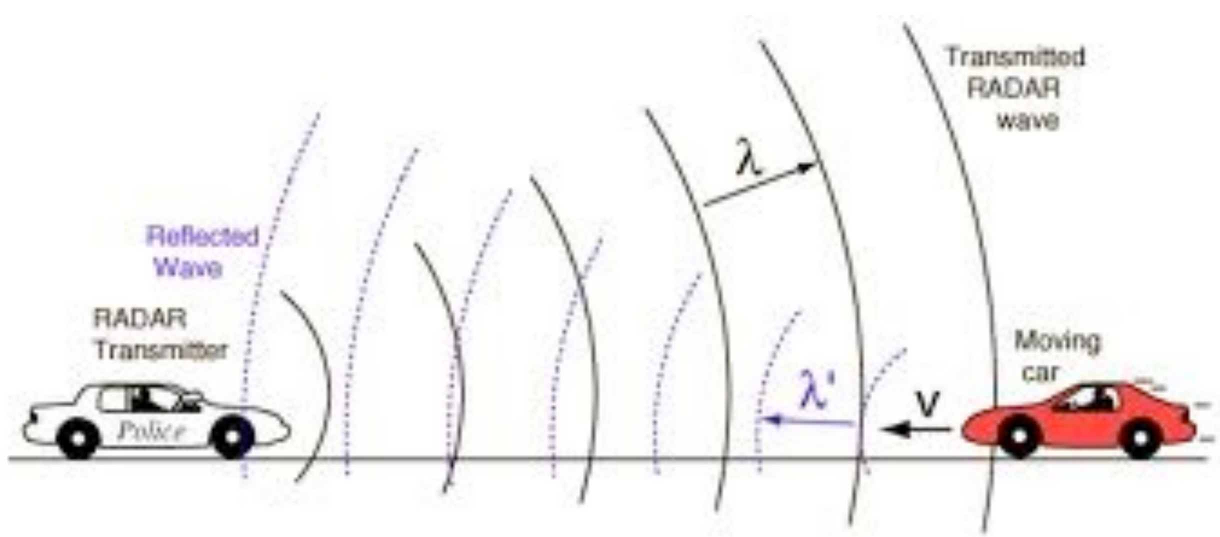
$$2A\cos(\frac{1}{2}(\omega_1 - \omega_2)t)$$

modulating wave envelope.

Doppler effect



You've all noticed it when a moving siren passes you, frequency goes from high to low.



Police radar speed traps also use Doppler's effect. So do bats and dolphins and whales, with their sonar.

Doppler effect

Case 1: $v_{source} = 0, \ v_{observer} \neq 0$

$$\lambda_{emit} = v_{sound}/f_{emit} = \lambda_{obs} = (v_{sound} - v_{observer})/f_{obs}$$

Both source and observer see same wavelength, but differing effective speed of sounds, because the observer sees sound's velocity relative to their own. So they see sound as moving slower if they're moving away from it, or faster if they're moving toward it.

Case 2: $v_{source} \neq 0, \ v_{observer} = 0$

$$\lambda_{obs} = \lambda_{emit} - (v_{source}/f_{emit}) = (v_{sound} - v_{source})/f_{emit}$$

Wavelength is stretched or compressed, depending on whether the source is moving away or toward the observer. Put these two cases together to get the general case (next slide).

Doppler, cont.

Put together: $$f_{observed} = \frac{v_{sound} - v_{observer}}{v_{sound} - v_{source}} f_{emitted}$$

Signs: replace - signs with +, depending on v directions.

if $v_{sound} \gg v_o, v_s$

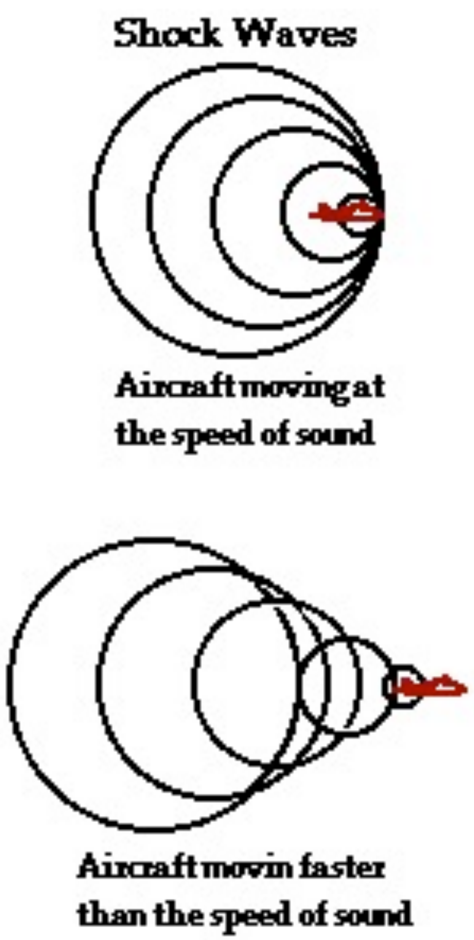
then $$\frac{v_{sound} - v_{observer}}{v_{sound} - v_{source}} \approx 1 - \frac{v_{observer} - v_{source}}{v_{sound}}$$

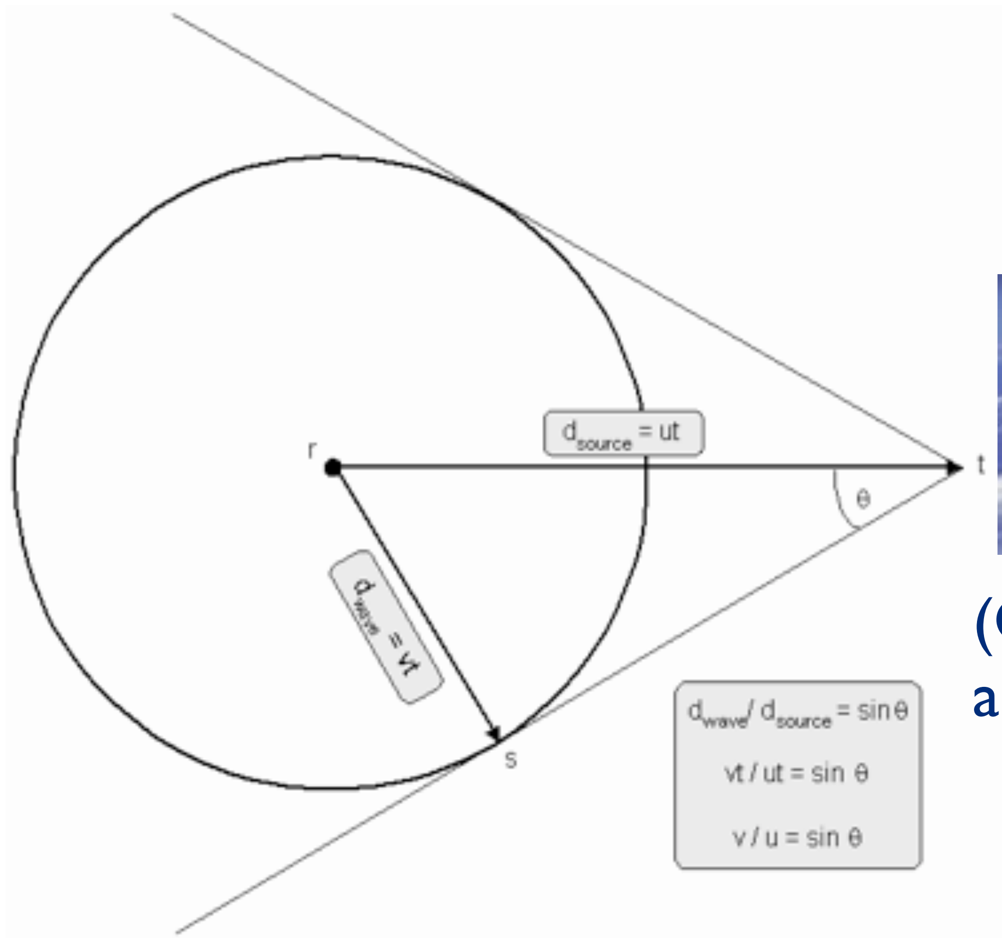
Aside (later), for light: $$f_{obs} = f_{source}\sqrt{\frac{1 \pm v_{rel}/c}{1 \mp v_{rel}/c}}$$

Top sign if source is coming at us (blueshift),
bottom sign if it's moving away (red).

Sonic boom!

If $\text{Mach number} = v_{object}/v_{sound}$ >1.





(Concorde, mach 2 around 1350 mph!)

$$\sin\theta = v_{sound}/v_{source} \quad = 1/\text{Mach number}$$