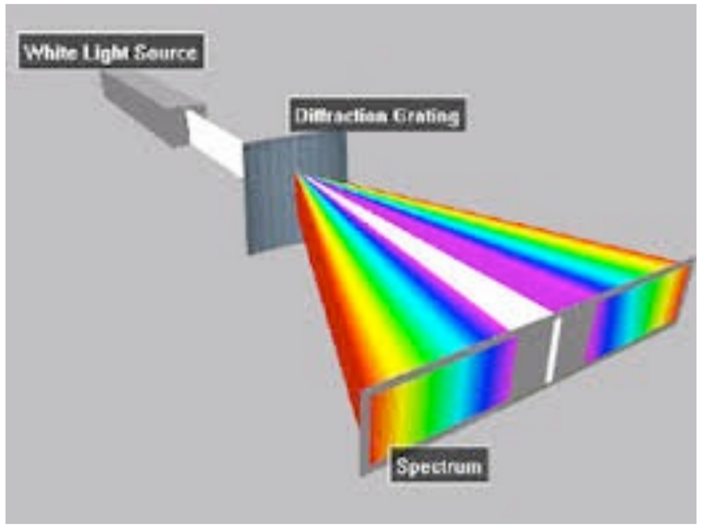
Diffraction*

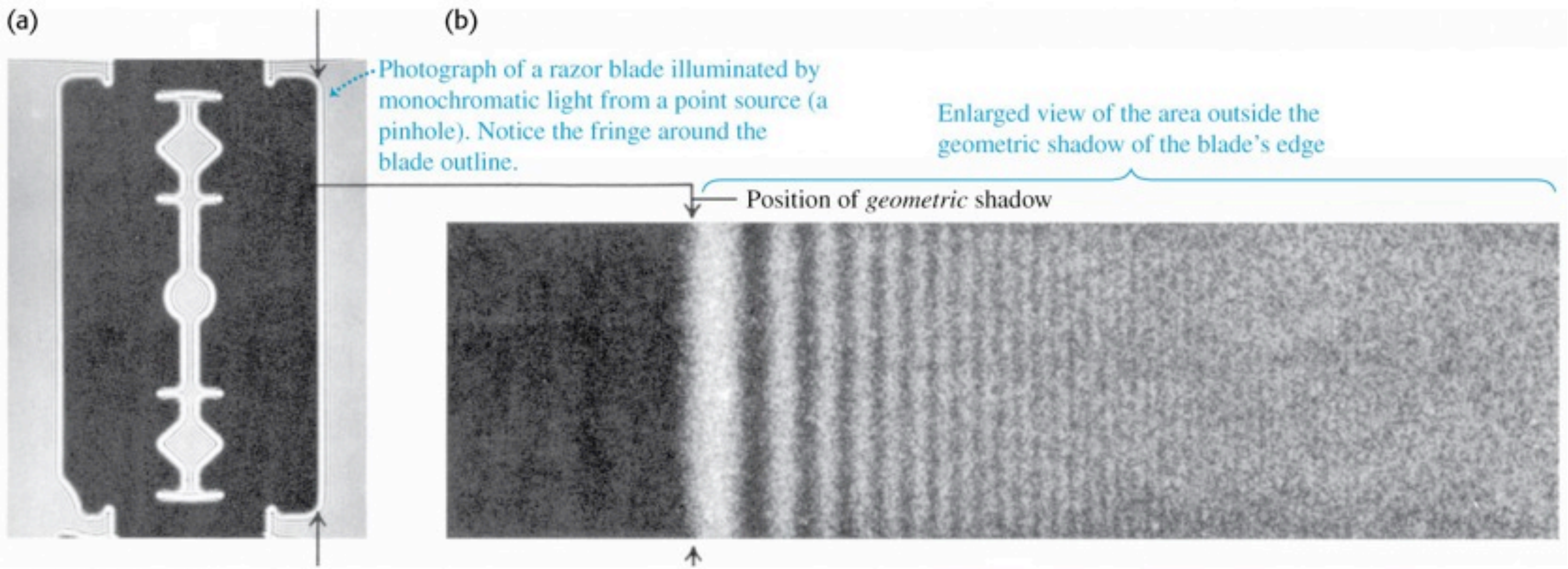
Ken Intriligator's week 10 lecture, Dec.3, 2013



* = various kinds of interference effects. For example why beams of light spread out. Colors on CDs. Diffraction gratings.

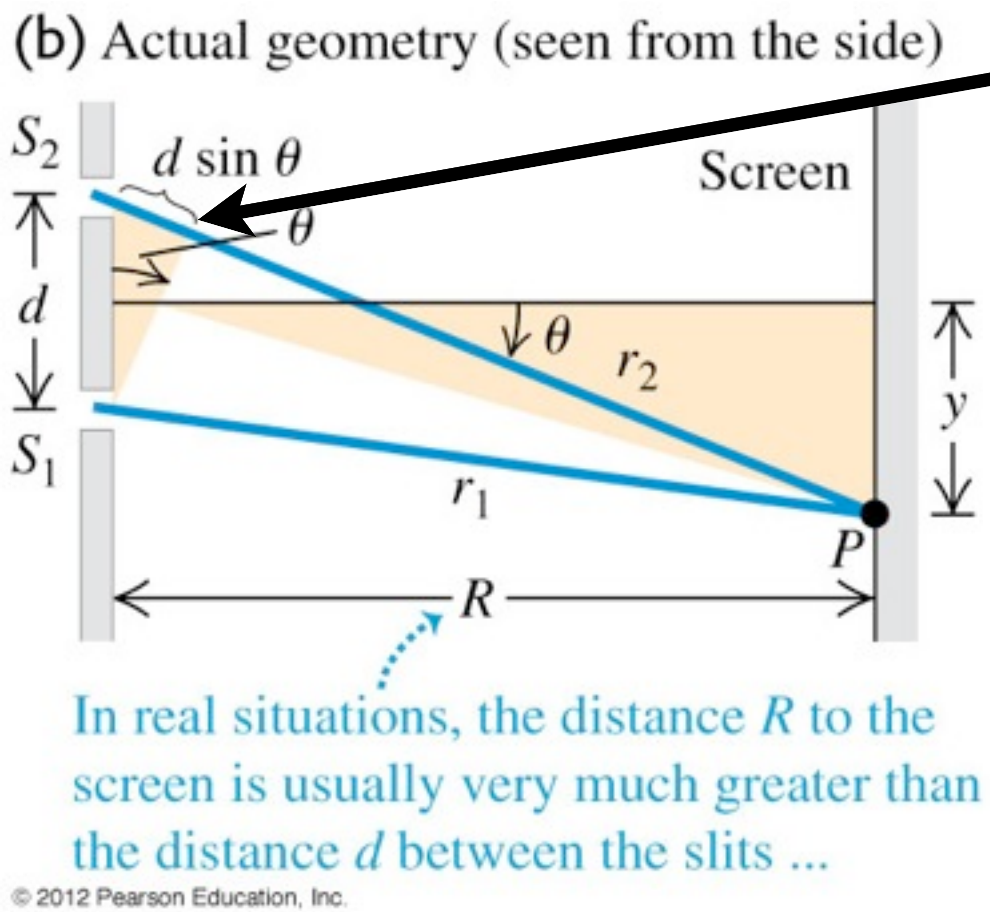
E.g. : Shadow edges

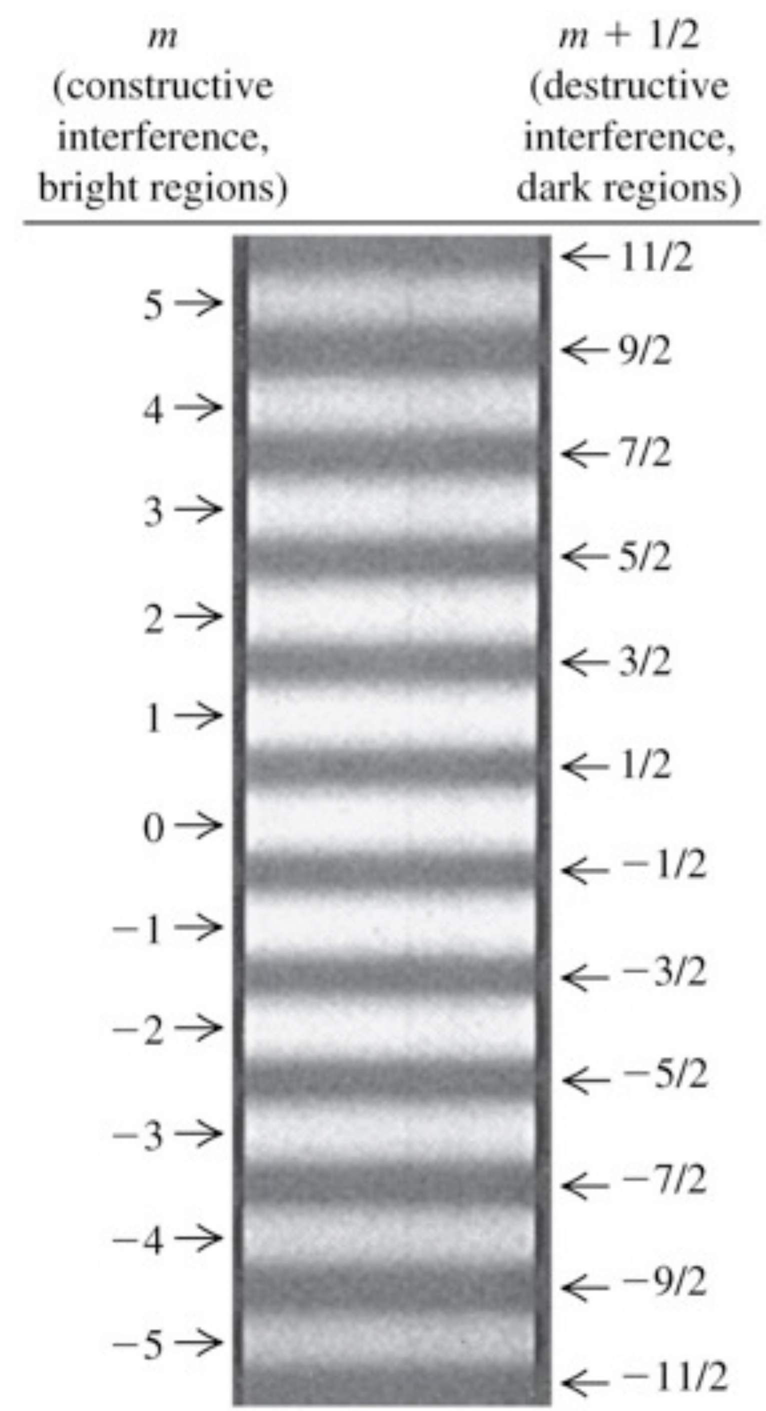
The edges of shadows can an interesting constructive / destructive interference pattern, related to the fact that light is a wave. Can see why from Huygen's principle: every point on light front acts as a point source for later light wave.



Recall 2slit interference

$$\Delta L \approx d \sin\theta$$





Bright: $d \sin\theta = m\lambda$

Dark: $d \sin\theta = (m + \frac{1}{2})\lambda$

Note: larger wavelength (redder) gives more spread-out fringes.

Recall 2-slit intensity

$$I \sim \langle E_{tot}^2 \rangle$$ (Time averaged)

trig identity: $$\cos a + \cos b = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$$

source 1 wave   source 2 wave

$$E_{tot}/E_0 = \cos(kL - \omega t) + \cos(kL' - \omega t) = 2\cos(\frac{k\Delta L}{2})\cos(kL_{ave} - \omega t)$$

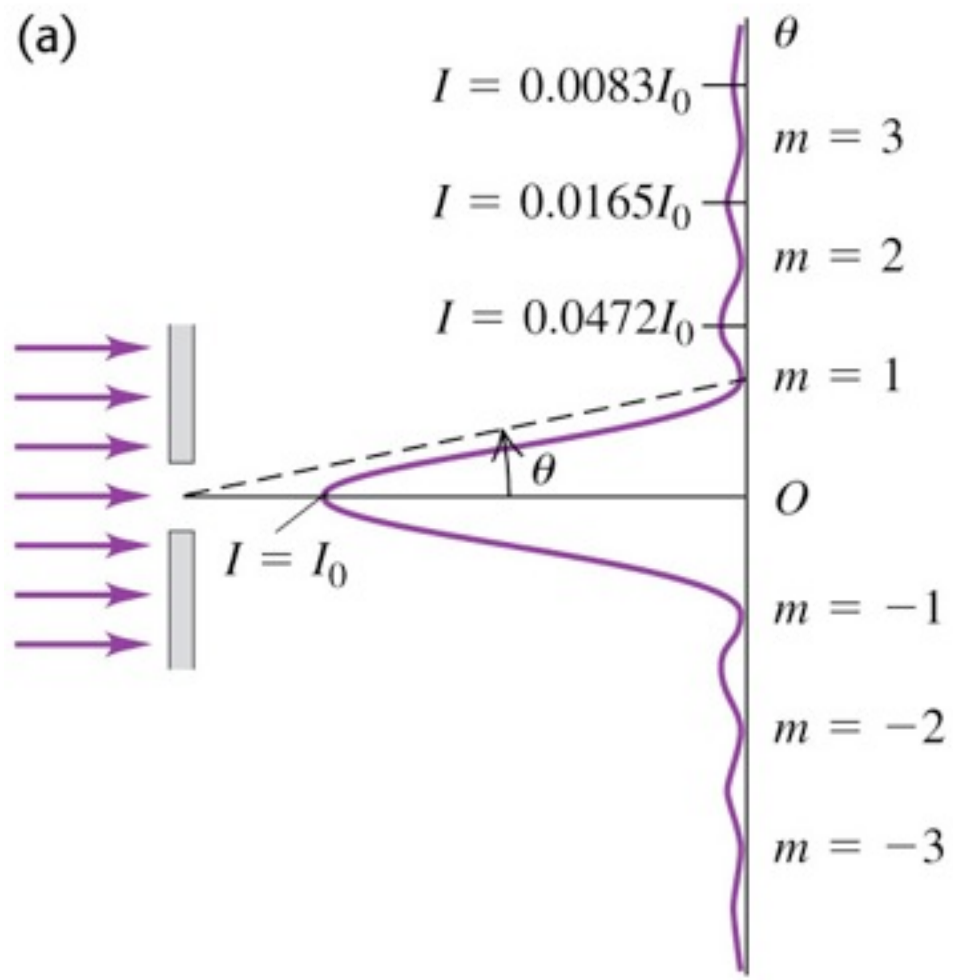
$$I_2 = 4I_1\cos^2(\frac{1}{2}k\Delta L) = 4I_1\cos^2(\pi\Delta L/\lambda)$$

Bright: $d\sin\theta = m\lambda$ **Dark:** $d\sin\theta = (m+\frac{1}{2})\lambda$

This was for 2 point sources = slits of approximately zero width (i.e. small width compared to light's wavelength). We'll now consider diffraction from a **single slit** of general width a.

Single slit diffraction

Consider barrier with width a. Observe interesting light patterns, projected on the screen, depending on the relative size of the slit width and the light's wavelength.



Demo: show change with a.

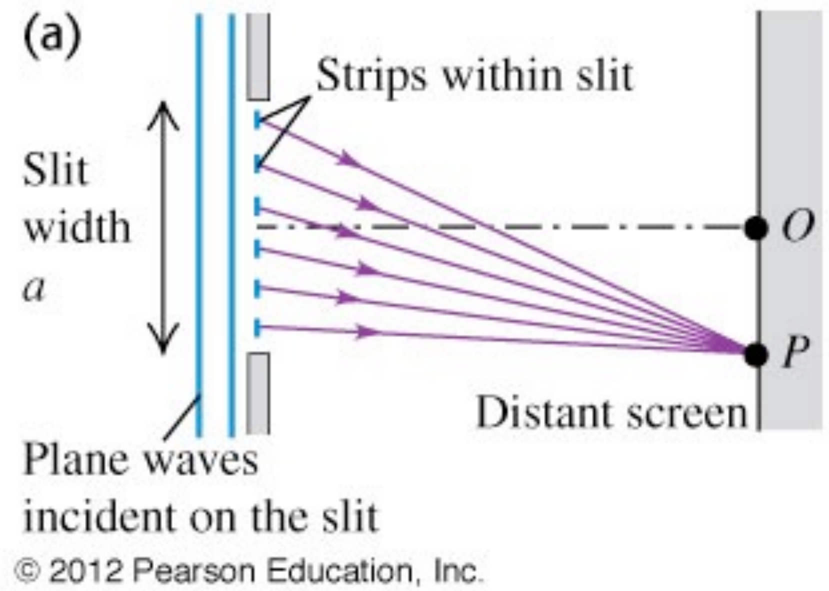
We'll now derive this from interference, thinking of each element of the slit as a point source and adding up their interference effect.

Single slit derivation

Consider interference from N point sources (tiny holes in the barrier), each separated by distance d:

Total:

$$E = A\sum_{i=1}^{N}\cos(kL_i - \omega t)$$



$$L_i = L_1 + (i-1)d\sin\theta$$

sum the series (using trig / tricks):

$$E = A\cos(kL_{ave} - \omega t)\frac{\sin(\frac{kNd\sin\theta}{2})}{\sin(\frac{kd\sin\theta}{2})}$$

Setting N=2 reduces to the 2-slit result from last week, by using trig identity: $\sin 2\beta = 2\sin\beta\cos\beta$

Constructive interference when $d\sin\theta = m\lambda$

Single slit deriv. cont.

$$E = A\cos(kL_{ave} - \omega t)\frac{\sin(\frac{kNd\sin\theta}{2})}{\sin(\frac{kd\sin\theta}{2})} \rightarrow I \sim E^2 = \delta I_0 \frac{\sin^2(\frac{\pi N d\sin\theta}{\lambda})}{\sin^2(\frac{\pi d\sin\theta}{\lambda})}$$

Now replace: $N \to \infty,\ d \to 0,\ Nd = a.$

So $\sin(\pi d\sin\theta/\lambda) \to \pi d\sin\theta/\lambda = \pi\frac{a}{N}\sin\theta/\lambda$

And call $N^2\delta I_0 \to I_0$

= Maximum intensity from N constructive sources

= N-squared times that of a single source.

Final result on next slide...

Single slit intensity

Single slit diffraction

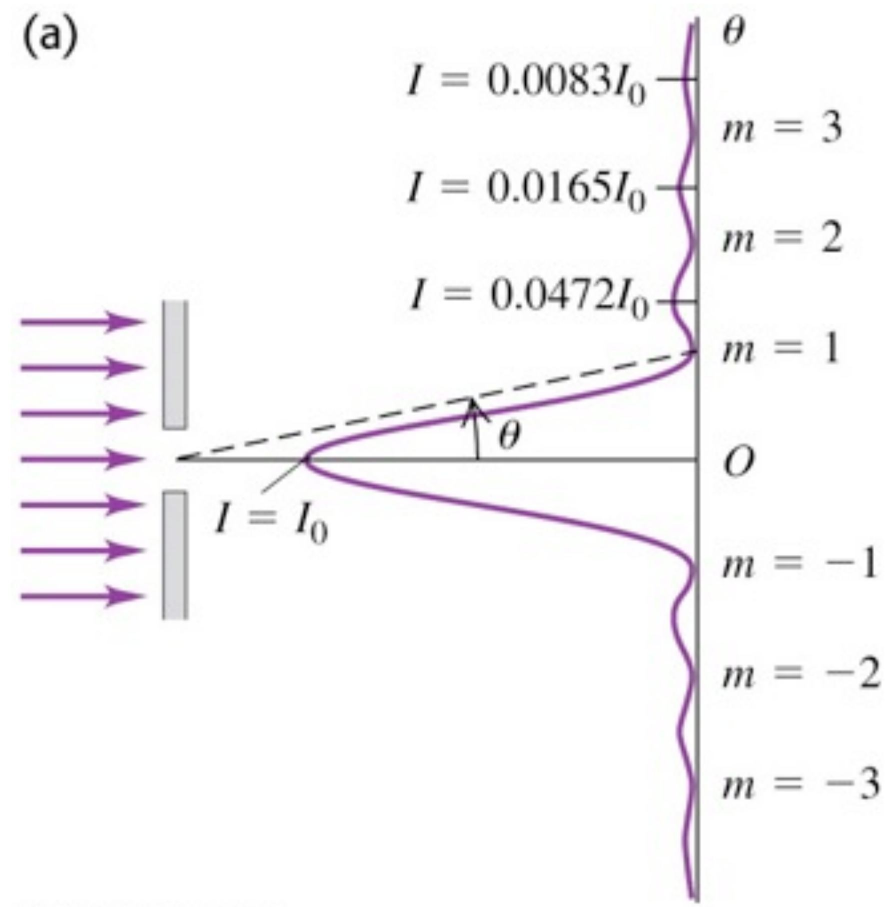
intensity result:

$$I \rightarrow I_0 \Big[\frac{\sin(\frac{\pi a \sin\theta}{\lambda})}{(\frac{\pi a \sin\theta}{\lambda})}\Big]^2$$

Maximum intensity, at

$\theta = 0$ The function of theta = one there.

The function of theta gives this shape for general angles theta:

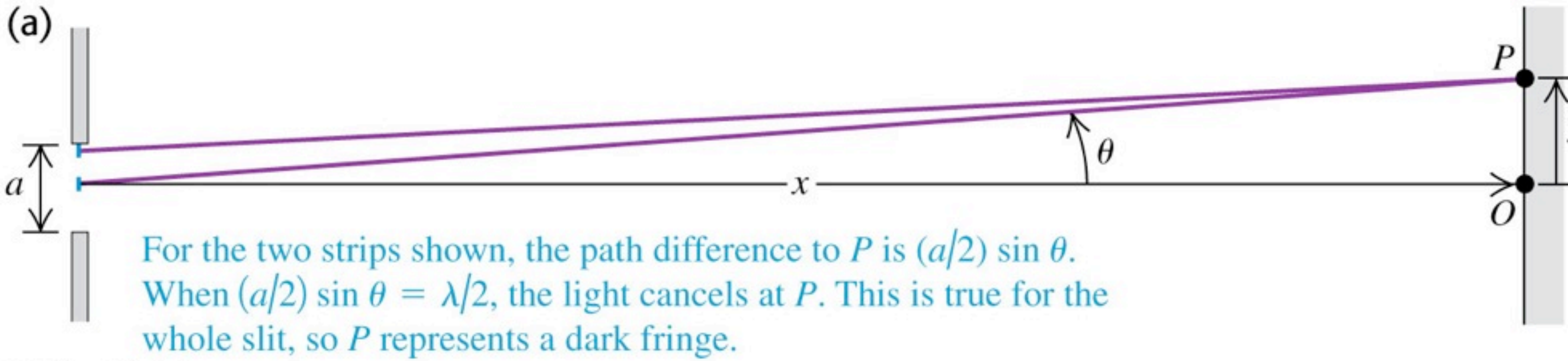


Zero intensity (dark fringes) at

$$\sin\theta = m\lambda/a, \;\; m = \pm 1, \pm 2, \ldots$$

Note larger wavelengths (redder) get wider pattern.
And for a=0, get constant intensity.

Beam spreading



First minimum of our intensity result (m=1) is at $\sin\theta = \lambda/a$

$\sin\theta \approx \tan\theta = \Delta y/D$ So $\Delta y \approx D\lambda/a$

$$\text{width} \approx \frac{D\lambda}{a} + a \approx \frac{D\lambda}{a}$$ for large D.

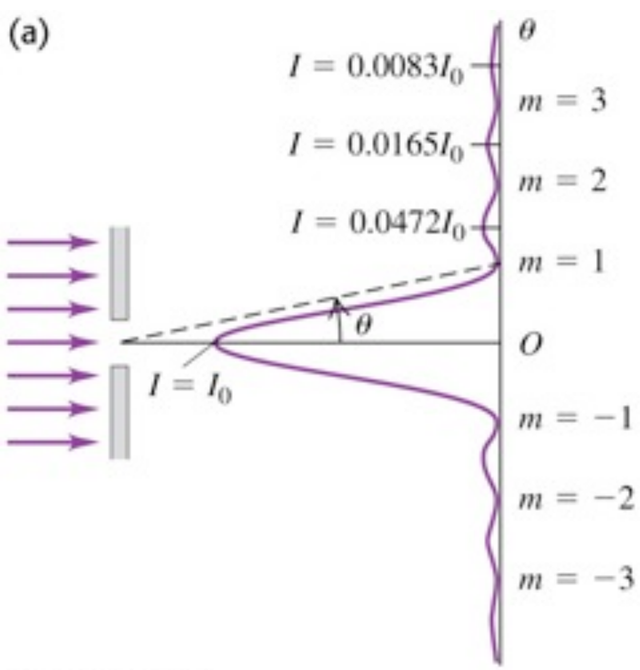
Aside: Related to property of Fourier transforms: $\Delta y \Delta k_y \geq 2\pi$

$$\Delta y = a$$
$$\Delta k_y = k\Delta\theta$$

Circular holes

Similar interference result, but the math is more complicated. Get some special functions (Bessel functions) beyond the scope of Physics 2C.

$$I = I_0[\frac{J_1(2\pi a \sin\theta/\lambda)}{2\pi a \sin\theta/\lambda}]^2$$



Function is different, but shape is qualitatively similar. First minimum at:

$$\sin\theta \approx 1.22\lambda/2R$$

R= radius of circular hole.

Resolvability

Raleigh: two spots are just barely resolvable if the central maximum of one is at the first minimum of the other. Their angular separation is then

$$\sin\theta_R = (1.22\lambda/d) \approx \theta_R$$

E.g. d is your pupil diameter and the angle is the minimum angle that you can visually resolve between stars. Bigger pupils (or telescope lenses) allow smaller angles to be resolved.

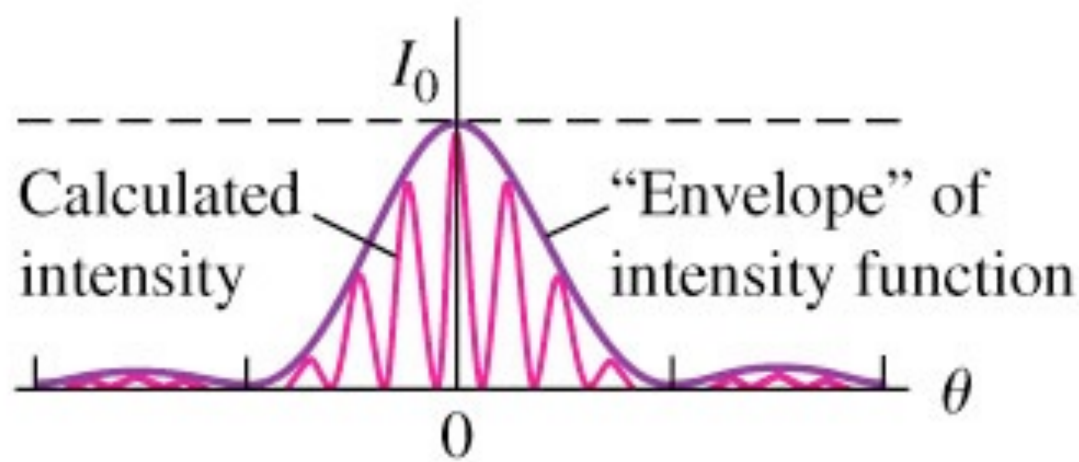
Two slits of width a

$$I = I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right) \left[\frac{\sin\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\frac{\pi a \sin\theta}{\lambda}}\right]^2$$

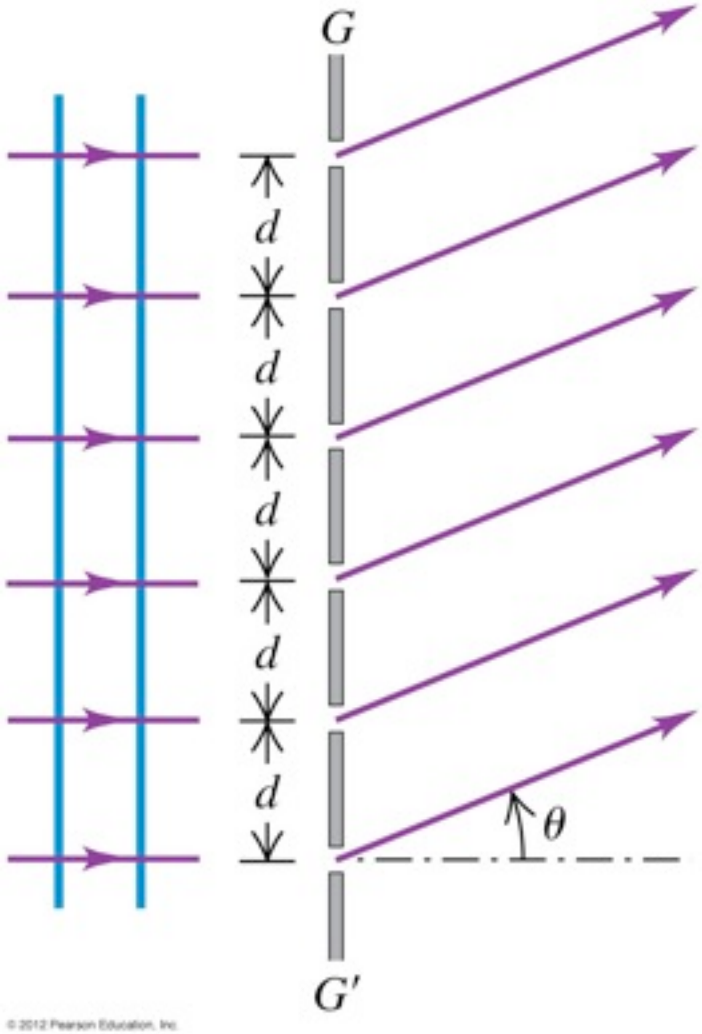
From the two slits.

From their width.

(c) Calculated intensity pattern for two slits of width a and separation $d = 4a$, including both interference and diffraction effects



Diffraction grating



Recall the N slit interference result:

$$E = A\cos(kL_{ave} - \omega t)\frac{\sin(\frac{kNd\sin\theta}{2})}{\sin(\frac{kd\sin\theta}{2})}$$

$$\rightarrow \quad I \sim E^2 = \delta I_0 \frac{\sin^2(\frac{\pi N d\sin\theta}{\lambda})}{\sin^2(\frac{\pi d\sin\theta}{\lambda})}$$

Constructive interference when: $d\sin\theta = m\lambda$

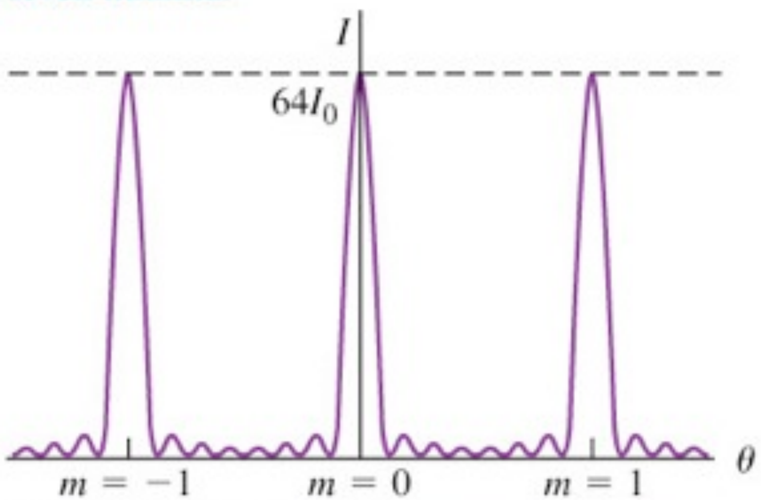
$$m = 0,\ \pm1,\ \pm2,\ \ldots$$

Increasing N makes the bright peaks taller and thinner.

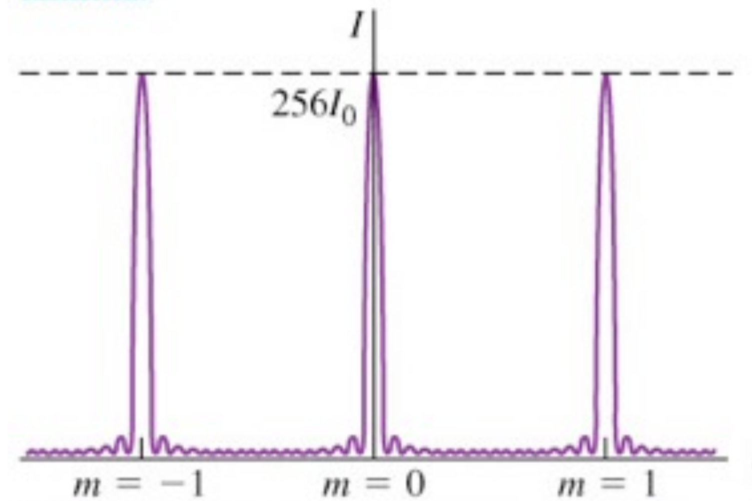
Diffraction grating cont.

$$I \sim E^2 = \delta I_0 \frac{\sin^2(\frac{\pi N d \sin\theta}{\lambda})}{\sin^2(\frac{\pi d \sin\theta}{\lambda})} \rightarrow \Delta\theta_{HW} = \lambda / N d \cos\theta$$

(b) $N = 8$: eight slits produce taller, narrower maxima in the same locations, separated by seven minima.



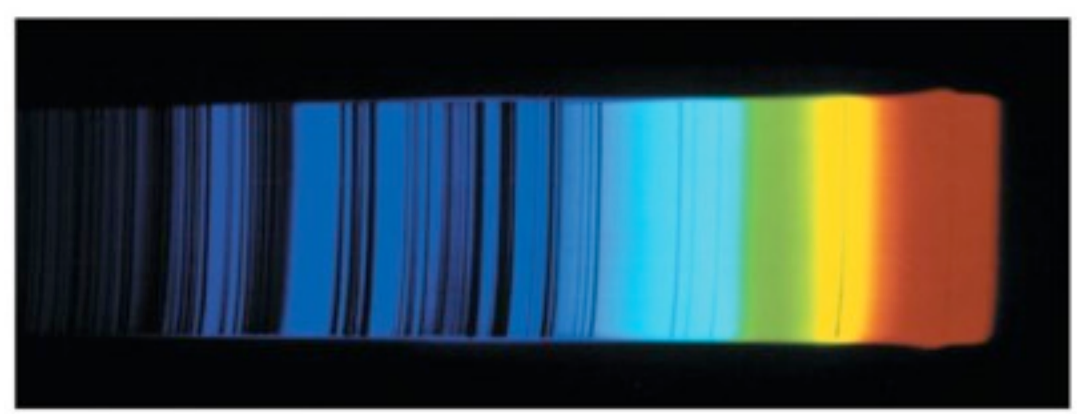
(c) $N = 16$: with 16 slits, the maxima are even taller and narrower, with more intervening minima.



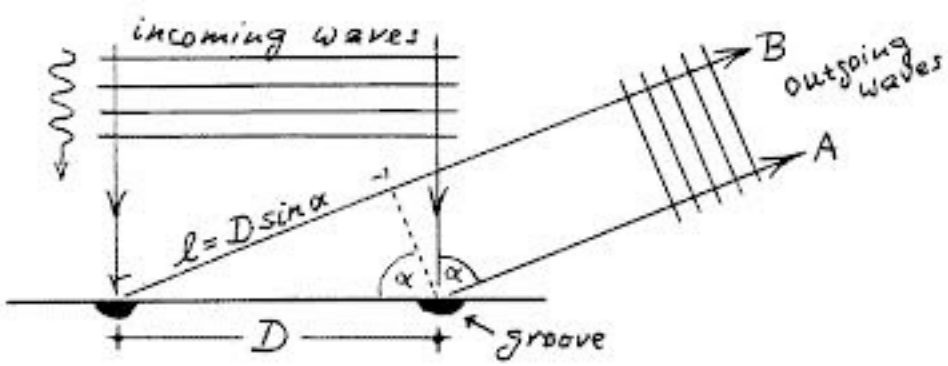
peaks @: $d \sin\theta = m\lambda$

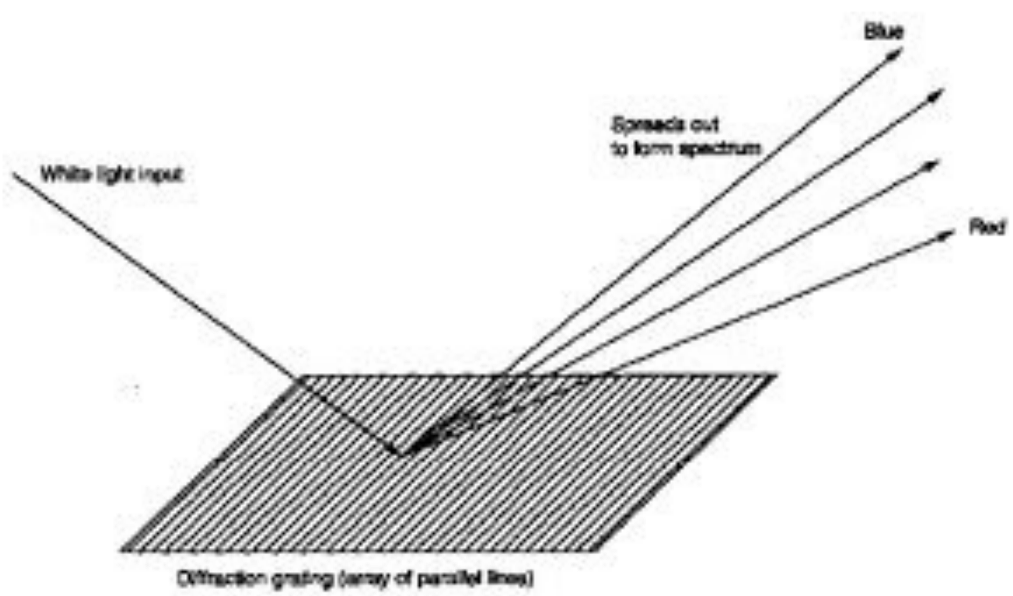
So redder light at bigger angles theta for e.g. m=1:

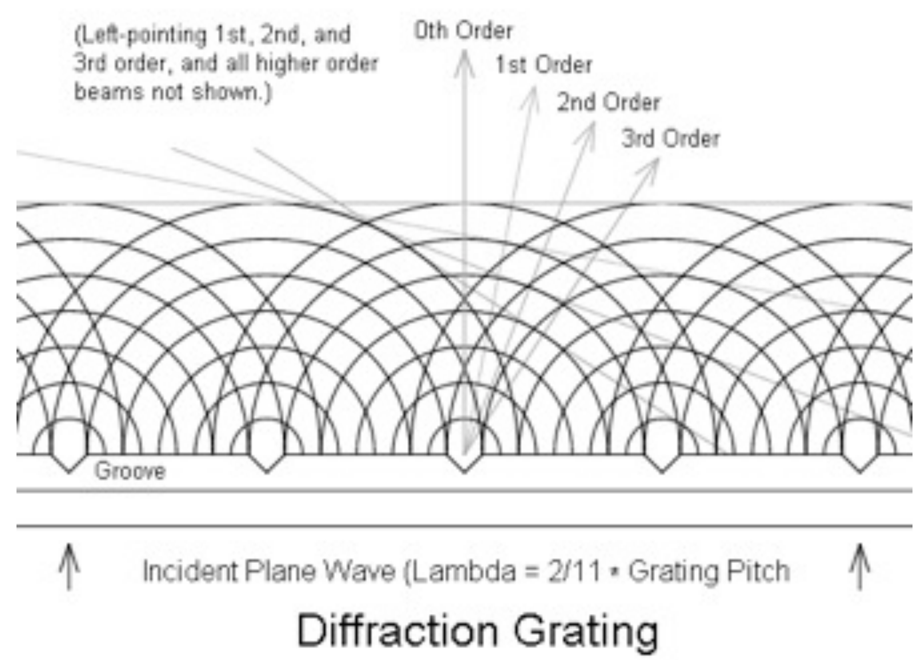
(b)



CD's reflection ~ a diffraction grating







Diffr. Grating's R-power

$R = \lambda/\Delta\lambda$ $\Delta\theta_{HW} = \lambda/Nd\cos\theta$

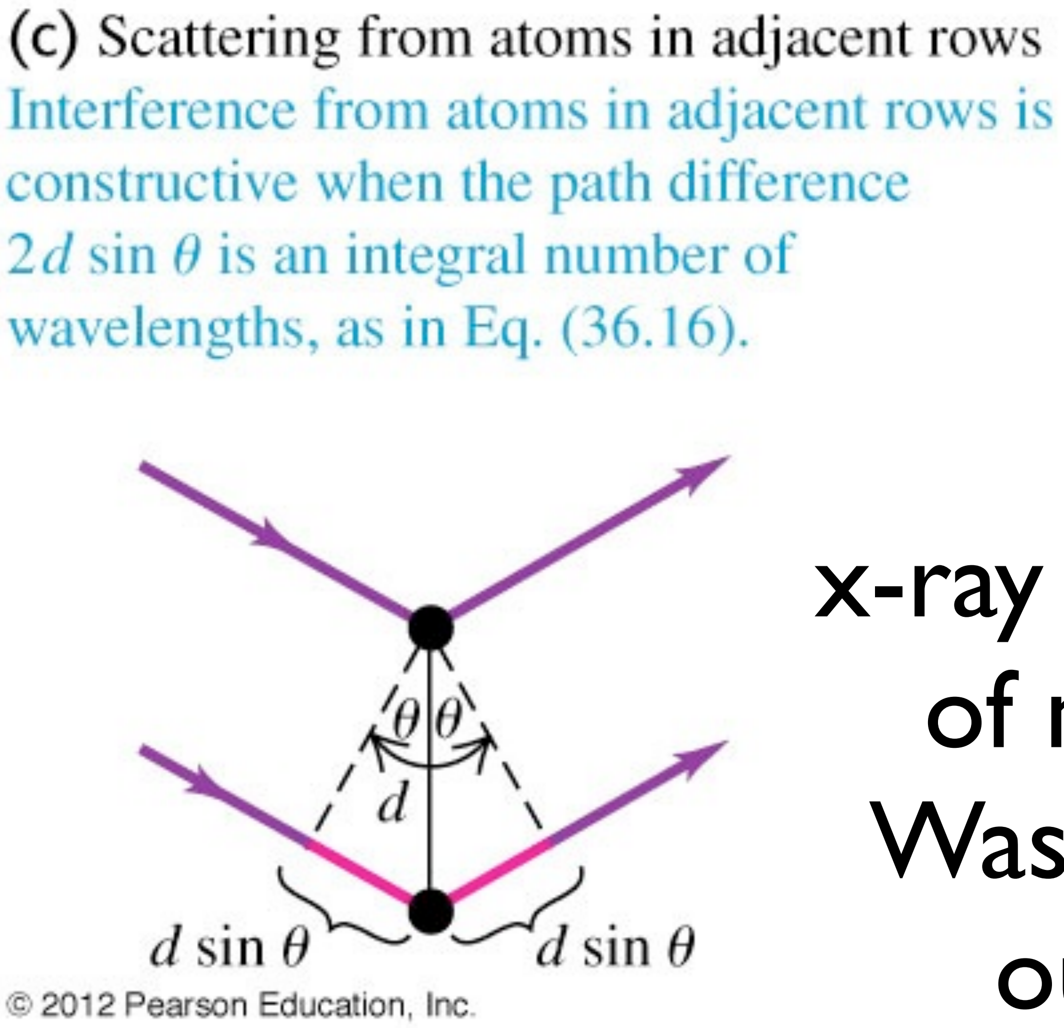
$d\cos\theta\Delta\theta = m\Delta\lambda$ → $R = Nm$

Can better resolve wavelengths for larger R, i.e. larger N and / or m.

Diffraction grating obs.:

Gasses of elements have a characteristic spectrum of quantized light frequencies / wavelengths (from quantum mechanics). Using a diffraction grating, can identify the glowing gas. Used to study stars' identity / characteristics. Also, see the effect of Doppler effect for light: distant stars are moving away, so their star's spectrum is red-shifted by the Doppler effect. Can be quantitatively measured using diffraction gratings, determines how fast distant starts are moving away from us. Measures the expansion of the Universe (Big Bang Cosmology / Inflation).

x-ray diffraction



constructive interference:

$$2d \sin \theta = m\lambda$$ Bragg peaks.

x-ray diffraction shows the structure of materials, crystals, DNA, etc.. Was & still is hugely important for our understanding of matter.

1927: Davisson & Germer found interference like this for electrons! Showed quantum wave-nature for matter.