10/24 Lecture outline

⋆ **Reading for today's lecture: Luke p. 65-80; Tong p. 35-41.**

• Continue computing some amplitudes in our nucleon + meson toy model, via

$$\langle f|(S-1)|i\rangle = \langle f|Te^{-i\int d^4x\mathcal{H}_I(x)}|i\rangle \equiv i\mathcal{A}_{fi}(2\pi)^4\delta^{(4)}(p_f - p_i).$$

Last time: meson decay $\phi \to N + \bar{N}$ has $\mathcal{A} = -g + O(g^3)$.

• Time out for a few comments. The scalar ϕ and coupling $g\phi\psi\bar{\psi}$ is also a good toy model for the Higgs' coupling to fermions. Here, and there, the scalar "Yukawa" coupling mediates a force. The strong, weak, and electromagnetic forces are communicated by spin 1 gauge fields. Gravity is mediated by the spin 2 graviton (and the difference between spin 1 vs spin 2 is part of why quantum gravity is conceptually and technically challenging). Spin 0 scalars can also mediate forces, as in this example. We'll see that their force is always attractive (even spins always lead to attractive forces). Fifth force experimental bounds strongly constrain the existence, mass, and couplings of fundamental scalars.

In our toy model, where ψ and $\bar{\psi}$ are scalars, the theory has a vacuum instability, since a cubic potential isn't bounded below. This shows up only indirectly in perturbation theory, and is more of a non-perturbative issue.

Let's also do some dimensional analysis. Recall that $[\phi] = 1$ and $[d^3k/2\omega] = 2$, so $[a(k)] = [a^\dagger(k)] = -1$. So $[|i,f\rangle] = -n_{i,f}$ and $[\mathcal{A}] = 4 - n_i - n_f$. In our example, $[g] = 1$, so $\mathcal{A}(\phi \to N + \bar{N}) = -g$ is dimensionally consistent. Good.

• Now consider $N + N \to N + N$, to $\mathcal{O}(g^2)$. The initial and final states are

$$|i\rangle = b^\dagger(p_1)b^\dagger(p_2)|0\rangle, \qquad \langle f| = \langle 0|b(p_1')b(p_2').$$

The term that contributes to scattering at $\mathcal{O}(g^2)$ is

$$T\frac{(-ig)^2}{2!}\int d^4x_1 d^4x_2 \phi(x_1)\psi^\dagger(x_1)\psi(x_1)\phi(x_2)\psi^\dagger(x_2)\psi(x_2).$$

The term that contributes to $S-1$ thus involves

$$\langle p_1'p_2'| :\psi^\dagger(x_1)\psi(x_1)\psi^\dagger(x_2)\psi(x_2): |p_1p_2\rangle = \langle p_1'p_2'| :\psi^\dagger(x_1)\psi^\dagger(x_2)|0\rangle\langle 0|\psi(x_1)\psi(x_2)|p_1,p_2\rangle.$$

$$= \left(e^{i(p_1'x_1+p_2'x_2)} + e^{i(p_1'x_2+p_2'x_1)}\right)\left(e^{-i(p_1x_1+p_2x_2)} + e^{-i(p_1x_2+p_2x_1)}\right).$$

The amplitude involves this times $D_F(x_1 - x_2)$ (from the contraction), with the prefactor and integrals as above. The final result is

$$i(-ig)^2 \left[\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_1 - p_2')^2 - \mu^2} \right] (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2').$$

Explicitly, in the CM frame, $p_1 = (\sqrt{p^2 + m^2}, p\widehat{e})$ and $p_2 = (\sqrt{p^2 + m^2}, -p\widehat{e})$, $p_1' = (\sqrt{p^2 + m^2}, p\widehat{e}')$, $p_2' = (\sqrt{p^2 + m^2}, -p\widehat{e}')$, where $\widehat{e} \cdot \widehat{e}' = \cos\theta$, and get

$$\mathcal{A} = g^2 \left(\frac{1}{2p^2(1 - \cos\theta) + \mu^2} + \frac{1}{2p^2(1 + \cos\theta) + \mu^2} \right).$$

According to the above, $[\mathcal{A}(2 \to 2)] = 0$ and the above is consistent with that. Good.

As we'll discuss, scattering by ϕ exchange leads to an attractive Yukawa potential.