10/3 Lecture outline

⋆ **Reading for today's lecture: Coleman to end of lecture 4 (p. 37).**

• Recall where we left off: $\langle x^\mu|0^\mu\rangle = \int \frac{d^3k}{(2\pi)^3} e^{-ip\cdot x}$ is not zero even for spacelike separation, $x^2 < 0$. We'll drop position eigenstates and operators.

• Multiparticle warmup: recall SHO, $[a, a^\dagger] = 1$, and states. Recall $\mathbf{1} = |0\rangle\langle 0| + \sum_{n=1}^\infty |n\rangle\langle n|$, sum over phonon occupation number states.

• QFT in $d = 3+1$ dimensions, replace particles with ripples of quantum field, e.g. $\phi(t, \vec{r})$. Mention QFT in $d = 0+1$ dimensions is QM, with $q(t)$ playing role of $\phi(t)$.

• Multiparticle states. E.g. $|\vec{k}_1, \vec{k}_2, \ldots \vec{k}_n\rangle$, with completeness

$$1 = |0\rangle\langle 0| + \sum_{n=1}^{\infty} \frac{1}{n!} \int \frac{d^3\vec{k}_1}{(2\pi)^3} \cdots \frac{d^3\vec{k}}{(2\pi)^3} |\vec{k}_1 \ldots \vec{k}_n\rangle\langle \vec{k}_1 \ldots \vec{k}_n|.$$

Introduce a box for the moment, to make momenta discrete. Can then count how many excitations of each momenta. Fock space description, like counting the excitation level of the SHO. Like, there, we'll introduce creation and annihilation operators.

• Classical and quantum particle mechanics, $L(q_a, \dot{q}_a, t)$, $p_a = \partial L/\partial \dot{q}_a$, $\dot{p}_a = \partial L/\partial q_a$, $H = \sum_a p_a \dot{q}_a - L$. Get quantum theory by replacing Poisson brackets with commutators, $[q_a(t), p_b(t)] = i\delta_{ab}$. Recall $O_H(t) = e^{iHt} O_S e^{-iHt}$ and $i\frac{d}{dt} O_H(t) = [O_H(t), H]$.

• Classical field theory. E.g. scalars $\phi_a(t, \vec{x})$, with $S = \int d^4x \mathcal{L}(\phi_a, \partial_\mu \phi_a)$. Then $\Pi_a^\mu = \partial\mathcal{L}/\partial(\partial_\mu \phi_a)$, and E.L. eqns $\partial\mathcal{L}/\partial\phi_a = \partial_\mu \Pi_a^\mu$. Define $\Pi_a \equiv \Pi_a^0$. $H = \int d^3x (\Pi\dot{\phi}_a - \mathcal{L}) = \int d^3x \mathcal{H}$.

• Example: $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2\phi^2)$, gives $\Pi = \dot{\phi}$ and $\dot{\Pi} = \nabla^2\phi - m^2\phi$, the Klein-Gordon equation: $(\partial^2 + m^2)\phi = 0$. Can't interpret ϕ as a probability wavefunction because of solutions $E = \pm\sqrt{\vec{p}^2 + m^2}$.

But we'll see that the KG equation is fine as a quantum field theory. As a classical field theory, write general classical solution as

$$\phi_{cl}(x) = \int \frac{d^3k}{(2\pi)^3 (2\omega(k))} [a_{cl}(k) e^{-ikx} + a_{cl}^*(k) e^{ikx}],$$

where $a_{cl}(k)$ are classical constants of integration, determined by the initial conditions.

• Canonical quantization: generalize QM by replacing $q_a(t) \to \phi_a(t, \vec{x})$. QM is like QFT in zero spatial dimensions, with the field playing role of position before:

$$[\phi_a(\vec{x}, t), \Pi_b(\vec{y}, t)] = i\delta_{ab}\delta^3(\vec{x} - \vec{y}) \quad \textit{(Equal time commutators).}$$

• Symmetries of $\mathcal{L}$ and Noether's theorem. If a variation $\delta\phi_a$ changes $\delta L = \partial_\mu F^\mu$, then it's a symmetry of the action and there is a conserved current: $j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)}\delta\phi_a - F^\mu$.

Example: $x^\mu \to x^\mu + \epsilon^\mu$, $\delta\phi_a = \epsilon^\nu \partial_\nu \phi_a$, $\delta\mathcal{L} = \epsilon^\nu \partial_\nu \mathcal{L}$ (assuming no explicit x dependence). Get $T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_a}\partial_\nu \phi_a - g_{\mu\nu}\mathcal{L}$. Stress energy tensor. Conserved charge is $P_\mu = \int d^3\vec{x} T_{\mu 0}$.

Another example: $\Lambda^\mu_\nu = \delta^\mu_\nu + \omega^\mu_\nu$, leads to conserved $M_{\mu\rho\sigma} = x_\mu T_{\rho\sigma} - x_\sigma T_{\rho\mu}$. Conserved charge is $M_{\rho\sigma} = \int d^3x M_{0\rho\sigma}$. Conserved angular momentum.