10/10 Lecture outline

⋆ **Reading for today's lecture: Taylor chapter 5. Also consult Arovas notes.**

• Damped, forced harmonic oscillator

$$\left(\frac{d^2}{dt^2} + 2\beta\frac{d}{dt} + \omega_0^2\right) x = f(t)$$

where $f(t) = F(t)/m$. Consider the case $f(t) = Ref_0 e^{-i\omega t}$, and $x(t) = Rez(t)$. The full solution is $x(t) = x_h(t) + x_p(t)$, where $x_h(t)$ is the solution without forcing, and x_p is the solution of the above eqn with forcing. Write $x_p = Rez_p$ and take $z_p = A(\omega)e^{-i\omega t}$ and find

$$A(\omega) = \frac{f_0}{\omega_0^2 - 2i\beta\omega - \omega^2}.$$

Note that

$$|A(\omega)| = \frac{|f_0|}{\sqrt{(\omega^2 - \omega_0^2)^2 + 4\beta^2\omega^2}},$$

$$\arg A(\omega) = \arg f_0 + \delta, \qquad \delta \equiv \tan^{-1}\left(\frac{2\beta\omega}{\omega_0^2 - \omega^2}\right).$$

The amplitude has its maximum at $\omega_R = \sqrt{\omega_0^2 - 2\beta^2}$. The phase shift δ goes from 0 to π as ω increases from below to above ω_0, going through $\pi/2$ out of phase at $\omega = \omega_0$.

The quality of the oscillator is $Q \equiv \omega_R/2\beta$. It is $\sim$ the number of cycles the oscillator makes in one decay time.

In the steady state, dropping the transient solution, we have $x = x_p(t)$. The power transmitted to the oscillator by the forcing function is $P = \frac{dW}{dt} = F(t)\dot{x}$. The average power is the same as the average of what's dissipated by friction, and given by

$$P_{ave} = m|f_0|^2 \frac{\beta\omega^2}{(\omega^2 - \omega_0^2)^2 + 4\beta^2\omega^2}$$

For $\beta = 0$ and forcing at $\omega = \omega_0$, get $z_p = \frac{1}{2\omega_0} f(\omega_0) ite^{-i\omega_0 t}$.

• LRC circuit (not on midterm): replace $F(t) \to V(t)$, applied voltage, $x \to q(t)$, charge on capacitor, $k \to 1/C$, $b \to R$, and $m \to L$.

• Fourier transform (not on midterm):

$$f(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \widehat{f}(\omega) e^{-i\omega t}.$$

Replace $f_0 \to \widehat{f}(\omega)/2\pi$ in above, use superposition. Get

$$z_p(\omega) = \widehat{G}(\omega)\widehat{f}(\omega), \qquad \widehat{G}(\omega) = \frac{1}{\omega_0^2 - 2i\beta\omega - \omega^2}.$$