10/7 Lecture outline

⋆ **Reading for today's lecture: Taylor chapter 5. Also consult Arovas notes.**

Damped harmonic oscillator

$$\left(\frac{d^2}{dt^2}+2\beta\frac{d}{dt}+\omega_0^2\right)x=0,$$

(with $\gamma = 2\beta m$).

Note that this is a linear equation, so superposition of solutions. Take $x(t) = Re z(t)$ with $z(t) = \sum_i C_i e^{-i\omega_i t}$ and get $\omega^2 + 2i\beta\omega - \omega_0^2$, so solution has $\omega = \omega_\pm = -i\beta \pm (\omega_0^2 - \beta^2)^{1/2}$. Then $x(t) = Re(C_+ e^{-i\omega_+ t} + C_- e^{-i\omega_- t})$.

Underdamped case: $\beta^2 < \omega_0^2$ has $x(t) = e^{-\beta t}(C\cos(\nu t) + D\sin(\nu t))$, where $\nu = \sqrt{\omega_0^2 - \beta^2}$. Oscillates before damping.

Overdamped case: $\beta^2 > \omega_0^2$) has $x(t) = e^{-\beta t}(C\cosh(\widetilde{n}ut) + D\sinh(\widetilde{\nu}t))$, where $\widetilde{\nu} = \sqrt{\beta^2 - \omega_0^2}$.

Critically damped case: $\omega_0^2 = \beta^2$ has $x(t) = Ce^{-\beta t} + Dte^{-\beta t}$. Oscillations damp out fastest in this case.