10/3 Lecture outline

⋆ **Reading for today's lecture: Taylor chapters 3, and 4. Also consult Arovas notes.**

• Conservation of energy. The kinetic energy is $T = \frac{1}{2}m\vec{v}^2$, so $dT = m\vec{v}\cdot d\vec{v} = md\vec{r}\cdot\frac{d\vec{v}}{dt} = \vec{F}\cdot d\vec{r}$, which is the work done by $\vec{F}$ over displacement $d\vec{r}$. Integrate it

$$T_2 - T_1 = \int_C \vec{F}\cdot d\vec{r} \equiv W(1 \to_C 2), \tag{1}$$

where C is some path connecting points 1 and 2. The notation is to emphasize that the answer can depend on the path C taken, and can generally differ for different paths connecting two points.

• *Conservative* forces have the property that $\vec{F} = \vec{F}(\vec{r})$, independent of $\vec{v}$, and that the integral above is always independent of the path C, depending only on the two endpoints. This latter condition is true if and only if $\nabla \times \vec{F} = 0$ (follows from Stokes). In this case, it is always possible (locally) to write $\vec{F} = -\nabla U(\vec{r})$, where $U(\vec{r})$ is the potential energy. N.B. the potential energy is also often called $V(\vec{r})$. For conservative forces, the equation (1) becomes

$$T_2 - T_1 = \int_C \vec{F}\cdot d\vec{r} = -\int_1^2 \nabla U(\vec{r})\cdot d\vec{r} = U(\vec{r}_1) - U(\vec{r}_2). \tag{2}$$

This can be written as **conservation of energy** $E = T + V$:

$$E_1 = T_1 + U_1 = E_2 = T_2 + U_2 = E, \tag{3}$$

where $U_1 = U(\vec{r}_1)$ and $U_2 = U(\vec{r}_2)$. The relation (3) holds for *any* point on a particle's trajectory when the force is conservative.

• Note on time dependent potentials: can have time dependent $\vec{F}(\vec{r},t) = -\nabla U(\vec{r},t)$, and then

$$dU = \nabla U\cdot d\vec{r} + \frac{\partial U}{\partial t}dt = -\vec{F}\cdot d\vec{r} + \frac{\partial U}{\partial t}dt,$$

from which it follows that $d(T+U) = \frac{\partial U}{\partial t}dt$, so *mechanical* energy $T+U$ is not conserved in this case (though total energy is still conserved – mechanical energy is converted into other types of energy).