9/30 Lecture outline

⋆ **Reading for today's lecture: Taylor chapters 3, and 4. Also consult Arovas notes.**

• Previous lecture: Particles labeled by i. Net force on particle i is $\vec{F}_i = \vec{F}_i^{ext} + \sum_{j\neq i}\vec{F}_{ij} = \frac{d}{dt}\vec{p}_i$. As discussed last time, the 3rd law is better phrased as conservation of momentum for $\vec{p}_{tot} = \sum_i \vec{p}_i$, or more generally:

$$\frac{d}{dt}\vec{p}_{tot} = \sum_i \left(\vec{F}_i^{ext} + \sum_{j\neq i}\vec{F}_{ij}\right) = \sum_i \vec{F}_i^{ext} = \vec{F}^{ext}.$$

So if $\vec{F}^{ext} = 0$, then the total momentum does not change in time – it is a conserved quantity. We'll see that conservation of momentum is related to a symmetry: translations in space. There are two other important conservation laws which we'll discuss now – energy and angular momentum – which are related to symmetries under translations in time, and rotations, respectively. Conservation of energy is extremely important and useful – it is often the simplest way to solve or start to solve a problem. Let's first discuss a bit conservation of angular momentum, and then discuss conservation of energy more fully.

• Brief timeout - rockets: $\vec{p}(t) = (M + dm)\vec{v}$, $\vec{p}(t + dt) = \vec{p}(t) + d\vec{p} = M(\vec{v} + d\vec{v}) + dm(\vec{v} + d\vec{v} + \vec{u})$, so $\frac{d\vec{p}}{dt} = M\frac{d\vec{v}}{dt} + \vec{u}\frac{dm}{dt}$. Taking $F_{ext} = 0$ and using $dm/dt = -dM/dt$, get $Md\vec{v} = \vec{u}dM$. For constant $\vec{u}$, integrates to $\vec{v}(t) - \vec{v}_0 = -\vec{u}\ln(M_0/M(t))$.

• The angular momentum of particle i is $\vec{L}_i = \vec{r}_i \times \vec{p}_i$, and $\frac{d}{dt}\vec{L}_i = \vec{r}_i \times \frac{d}{dt}\vec{p}_i \equiv \vec{\tau}_i$, where we dropped a term which is zero and $\vec{\tau}_i$ is the torque exerted on particle i. Defining $\vec{L}_{tot} = \sum_i \vec{L}_i$, we then have

$$\frac{d}{dt}\vec{L}_{tot} = \sum_i \vec{r}_i \times \vec{F}_i = \sum_i \vec{r}_i \times \vec{F}_i^{ext} + 0 = \vec{\tau}^{ext},$$

where the internal torques cancel, much as with conservation of momentum:

$$0 = \sum_i \sum_{j\neq i} \vec{r}_i \times \vec{F}_{ij} = \sum_i \sum_{j>i} (\vec{r}_i - \vec{r}_j) \times \vec{F}_{ij},$$

which vanishes for central forces. Conservation of angular momentum is more general than this derivation – it is always true at a fundamental level, and related to symmetry under rotations. As mentioned last time, in E & M the $\vec{E}$ and $\vec{B}$ fields themselves can carry momentum. Likewise, they can carry angular momentum. Conservation of angular momentum will be discussed more fully later.