9/28 Lecture outline

$\star$ **Reading for today's lecture: Taylor chapters 3, and 4. Also consult Arovas notes.**

• Last time: Integrate the equation $\vec{F} = m\vec{a}$ for some simple examples, emphasizing that it is a second order differential equation, so there are two integration constants or boundary conditions – e.g. the position and velocity at $t = 0$, which then determine the solution for all t.

Examples of drag force, $F = F(v)$, friction force that opposes the motion. Integrate to find $v(t)$ via $\int dv/F(v) = \int dt/m$. Terminal velocity, $F(v_{term}) = 0$. Integrate $v(t)$ again to find $x(t)$ via $\int dx = \int dtv$.

• $F = -mg$ yields $z = -\frac{1}{2}gt^2 + v_0 t + z_0$; $F = -mg - bv$ yields $v(t) = v_0 e^{-bt/m} + v_\infty(1 - e^{-bt/m})$, with $v_\infty = -mg/b$ the terminal velocity (the minus sign means it's downward). Integrating again yields $z = z_0 + \frac{m}{b}(v_0 + \frac{mg}{b})(1 - e^{-bt/m}) - \frac{mg}{b}t$; $F = -mg - cv^2 \text{sign}(v)$ yields $v(t) = -\sqrt{mg/c}\tanh(gt/\sqrt{mg/c})$, where as chose $t = 0$ to be when $v = 0$ and now $v_\infty = -\sqrt{mg/c}$, and integrate again to get $z = z_0 - (v_\infty^2/g)\ln\cosh(gt/v_\infty)$.

• Some physics of air resistance. $\vec{F}_{drag} = -f(v)\hat{v}$. For low speeds $f(v) \approx bv + cv^2 = f_{lin} + f_{quad}$. The linear term comes from the viscosity η of the fluid, and is related to the length of the object as $b \sim \eta D$. The quadratic term comes from pushing the fluid out of the way, so it's proportional to the fluid density and the objects cross section area: $c \sim \rho D^2 C_{drag}$. Check units e.g. η comes from force per area $\sim \eta\frac{\partial v}{\partial y}$, and C_{drag} is dimensionless.

Examples: in air at STP $b = \beta D$ with $\beta = 1.6 \times 10^{-4} Ns/m^2$ and $c = \gamma D^2$ with $\gamma = 0.25 Ns^2/m^4$. Find $f_{quad}/f_{lin} = (1.6 \times 10^3 s/m^2)Dv$, so f_{quad} dominates for sufficiently large or fast objects, and f_{lin} dominates for sufficiently small or slow objects. Baseball ($D = .07m$, $v = 5m/s$) has $f_{quad}/f_{lin} \approx 600$, raindrop ($D = 10^{-3}m$, $v = 0.6m/s$) has $f_{quad}/f_{lin} \approx 1$, and Millikan oil drop ($D = 1.5 \times 10^{-6}m$, $v = 5 \times 10^{-5}m/s$ has $f_{quad}/f_{lin} \approx 10^{-7}$.

• Aside on dimensional analysis. Back of the envelope calculations. Example of Taylor's use of Life Magazine pictures to find energy yield of expanding nuclear fireball (quoted from Goodstein, States of Matter): $r = r(E, \rho, t) \sim (Et^2/\rho)^{1/5}$.

• Aside: vectors and fun with polar coordinates: $\dot{\hat{r}} = \dot{\phi}\hat{\phi}$, $\dot{\hat{\phi}} = -\dot{\phi}\hat{r}$, expressions for $\vec{r}$, $\vec{v}$, $\vec{a}$. You might want to review these things for later.

• Another example: $\vec{F} = q\vec{v} \times \vec{B}$, $V = v_x + iv_y$ has $\dot{V} = -i\omega V$, so $V = V_0 e^{-i\omega t}$ and $X = x + iy = X_0 e^{-i\omega t}$ with $\omega_c = qB/m$. Practice with complex numbers.