9/28 Lecture outline

⋆ **Reading for this week's lectures: Taylor chapters 2, 3, and 4.1 to 4.4. Also consult Arovas notes.**

• Last time $\vec{F}_1(\vec{r}_1) = -Gm_1m_2(\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|^3$. Leads to $\frac{d^2\vec{r}}{dt^2} = \vec{g}$ or $\frac{d^2}{dt^2}\vec{r}_1 = -Gm_2(\vec{r}_1 - \vec{r}_2)/|\vec{r}_1 - \vec{r}_2|^3$, respectively. Again, inertial and gravitational mass, person in a free falling elevator, Einstein's principle of equivalence. Tom Murphy's experiment.

• Other examples of forces: $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$, $\vec{F} = -b\vec{v}$, etc. Adding forces. Multiple particles.

• Third law, $\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}$. Example of gravity again.

More generally, relation with conservation of momentum. Particles labeled by $i$. Net force on particle $i$ is $\vec{F}_i = \vec{F}_i^{ext} + \sum_{j\neq i}\vec{F}_{ij} = \frac{d}{dt}\vec{p}_i$. The 3rd law is better phrased as conservation of momentum for $\vec{p}_{tot} = \sum_i \vec{p}_i$, or more generally:

$$\frac{d}{dt}\vec{p}_{tot} = \sum_i \left(\vec{F}_i^{ext} + \sum_{j\neq i}\vec{F}_{ij}\right) = \sum_i \vec{F}_i^{ext} = \vec{F}^{ext}.$$

So if $\vec{F}_{ext} = 0$, then the total momentum does not change in time – it is a conserved quantity. We'll see that conservation of momentum is related to a symmetry: translations in space.

Example from E&M where forces aren't equal and opposite, but momentum is still conserved. Energy conservation, and relation to time translation invariance.

• Integrate the equation $\vec{F} = m\vec{a}$ for some simple examples, emphasizing that it is a second order differential equation, so there are two integration constants or boundary conditions – e.g. the position and velocity at $t = 0$, which then determine the solution for all $t$. Continue with the examples, illustrating it first for motion in 1d:

$F = -mg$ yields $z = -\frac{1}{2}gt^2 + v_0t + z_0$; $F = -mg - bv$ yields $v(t) = v_0e^{-bt/m} + v_\infty(1 - e^{-bt/m})$, with $v_\infty = -mg/b$ the terminal velocity (the minus sign means it's downward). Integrating again yields $z = z_0 + \frac{m}{b}(v_0 + \frac{mg}{b})(1 - e^{-bt/m}) - \frac{mg}{b}t$; $F = -cv^2\text{sign}(v)$ yields $v(t) = -\sqrt{mg/c}\tanh(gt/\sqrt{mg/c})$, where as chose $t = 0$ to be when $v = 0$ and now $v_\infty = -\sqrt{mg/c}$, and integrate again to get $z = z_0 - (v_\infty^2/g)\ln\cosh(gt/v_\infty)$.