10/14 Lecture outline

⋆ **Reading: Luke, chapter 5**

• Last time, Dyson's formula: $S = Te^{-i\int d^4x \mathcal{H}_{int}}$. Use this to compute $\langle f|S-1|i\rangle$, using Wick's theorem,

$$T(\phi_1 \dots \phi_n) =: \phi_1 \dots \phi_n : + \sum_{contractions} : \phi_1 \dots \phi_n :$$

to eliminate the T ordering

• Illustrate this for our simple example of interacting theory:

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi^2 - \mu^2\phi^2) + (\partial\psi^\dagger\partial\psi - m^2\psi^\dagger\psi) - g\phi\psi\psi^\dagger.$$

Toy model for interacting nucleons and mesons. Treat last term as a perturbation.

Recall $\phi \sim a + a^\dagger$ for "mesons," $\psi \sim b + c^\dagger$, and $\psi^\dagger \sim b^\dagger + c$. We'll say that b annihilates a nucleon N and $c^\dagger$ creates an anti-nucleon $\bar{N}$. Conservation law, conserved charge $Q = N_b - N_c$. Examples of states:

$$|\phi(p)\rangle = a^\dagger(p)|0\rangle, \qquad |N(p)\rangle = b^\dagger(p)|0\rangle, \qquad |\bar{N}(p) = c^\dagger(p)|0\rangle.$$

Note then e.g.

$$\langle 0|\phi(x)|\phi(p)\rangle = e^{-ip\cdot x}, \qquad \langle 0|\psi(x)|N(p)\rangle = e^{-ip\cdot x}, \qquad \langle 0|\psi^\dagger(x)|N(p)\rangle = 0.$$

Example: meson decay. $|i\rangle = a^\dagger(p)|0\rangle$, $|f\rangle = b^\dagger(q_1)c^\dagger(q_2)|0\rangle$. Compute $\langle f|S|i\rangle = -ig\delta^4(p - q_1 - q_2)$ to $\mathcal{O}(g)$.

Now consider $N + N \to N + N$, to $\mathcal{O}(g^2)$. The initial and final states are

$$|i\rangle = b^\dagger(p_1)b^\dagger(p_2)|0\rangle, \qquad \langle f| = \langle 0|b(p_1')b(p_2').$$

The term that contributes to scattering at $\mathcal{O}(g^2)$ is

$$T\frac{(-ig)^2}{2!}\int d^4x_1 d^4x_2 \phi(x_1)\psi^\dagger(x_1)\psi(x_1)\phi(x_2)\psi^\dagger(x_2)\psi(x_2).$$

The term that contributes to $S-1$ thus involves

$$\langle p_1'p_2'| : \psi^\dagger(x_1)\psi(x_1)\psi^\dagger(x_2)\psi(x_2) : |p_1p_2\rangle = \langle p_1'p_2'| : \psi^\dagger(x_1)\psi^\dagger(x_2)|0\rangle\langle 0|\psi(x_1)\psi(x_2)|p_1, p_2\rangle.$$

$$= \left(e^{i(p_1' x_1 + p_2' x_2)} + e^{i(p_1' x_2 + p_2' x_1)}\right)\left(e^{-i(p_1 x_1 + p_2 x_2)} + e^{-i(p_1 x_2 + p_2 x_1)}\right).$$

The amplitude involves this times $D_F(x_1 - x_2)$ (from the contraction), with the prefactor and integrals as above. The final result is

$$i(-ig)^2 \left[\frac{1}{(p_1 - p_1')^2 - \mu^2} + \frac{1}{(p_1 - p_2')^2 - \mu^2}\right](2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_1' - p_2').$$

Explicitly, in the CM frame, $p_1 = (\sqrt{p^2 + m^2}, e\widehat{e})$ and $p_2 = (\sqrt{p^2 + m^2}, -p\widehat{e})$, $p_1' = (\sqrt{p^2 + m^2}, p\widehat{e}')$, $p_2' = (\sqrt{p^2 + m^2}, -p\widehat{e}')$, where $\widehat{e} \cdot \widehat{e}' = \cos\theta$, and get

$$\mathcal{A} = g^2 \left(\frac{1}{2p^2(1 - \cos\theta) + \mu^2} + \frac{1}{2p^2(1 + \cos\theta) + \mu^2}\right).$$