11/23 Lecture outline

⋆ **Reading: Luke chapter 10. Tong chapter 5**

• Do perturbation theory as before, but account for Fermi statistics, e.g. $T(\psi(x_1)\psi(x_2)) = -T(\psi(x_2)\psi(x_1))$ and likewise for normal ordered products. Consider in particular the propagator

$$\{\psi(x), \bar{\psi}(y)\} = (i\partial\!\!\!/_x + m)(D(x-y) - D(y-x)).$$

and the contraction

$$\langle 0|T(\psi(x)\bar{\psi}(y))|0\rangle = \int \frac{d^4p}{(2\pi)^4} \frac{i(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon} e^{-ip(x-y)}.$$

Vanishes for spacelike separated points. The momentum space fermion propagator is

$$\frac{i}{p\!\!\!/ - m + i\epsilon}.$$

Let's call the particle states nucleons and anti-nucleons (we could also call them electrons and positrons etc):

$$|N(p,r)\rangle = b(p)^{r\dagger}|0\rangle \qquad |\bar{N}(p,r)\rangle = c^{r\dagger}(p)|0\rangle.$$

Then

$$\langle 0|\psi(x)|N(p,r)\rangle = e^{-ipx}u^r(p), \qquad \langle N(p,r)|\bar{\psi}(x)|0\rangle = e^{ipx}\bar{u}^r(p).$$

Incoming fermions get a factor of $u^r(p)$, outgoing fermions gt $\bar{u}^r(p)$; incoming antifermions gets $\bar{v}^r(p)$, and outgoing antifermions get $v^r(p)$.

Write the amplitude by following the arrows backwards, from the head to the tail.

• Example, redo our meson-nucleon toy model, but now treating the nucleons as fermions, interacting with the scalar via $\mathcal{L}_{int} = -g\phi\bar{\psi}_a\Gamma_{ab}\psi_b(x)$. Compute various scattering amplitudes.

$N + \phi \to N + \phi$:

$$i\mathcal{A} = (-ig)^2\bar{u}^{r'}(p')\Gamma\left(\frac{i(p\!\!\!/ + q\!\!\!/ + m)}{(p+q)^2 - m^2 + i\epsilon} + \frac{i(p\!\!\!/ - q\!\!\!/' + m)}{(p-q')^2 - m^2 + i\epsilon}\right)\Gamma u^r(p).$$

$\bar{N} + \phi \to \bar{N} + \phi$:

$$i\mathcal{A} = -(-ig)^2\bar{v}^r(p)\Gamma\left(\frac{i(-p\!\!\!/ - q\!\!\!/ + m)}{(p+q)^2 - m^2 + i\epsilon} + \frac{i(-p\!\!\!/ + q\!\!\!/' + m)}{(p-q')^2 - m^2 + i\epsilon}\right)\Gamma v^{r'}(p').$$

$N + N \to N + N$:

$N + \bar{N} \to \phi + \phi$:

$N + \bar{N} \to N + \bar{N}$:

$\phi + \phi \to \phi + \phi$ (loop amplitude):

• Minus sign of fermion loop.

• Attractive Yukawa potential for both $\psi\psi \to \psi\psi$, and also $\psi\bar{\psi} \to \psi\bar{\psi}$. Recall $\mathcal{A}_{NR} = -i\int d^3\vec{r}e^{-i(\vec{p}'-\vec{p})\cdot\vec{r}}U(\vec{r})$. For $\psi\psi \to \psi\psi$, $\mathcal{A}_{NR} \supset -i(-ig)^2(2m)\frac{1}{(\vec{p}-\vec{p}')^2+\mu^2}$ when the spins are unchanged. Gives $U(\vec{r}) = -g^2e^{-\mu r}/4\pi r$. For $\psi\bar{\psi} \to \psi\bar{\psi}$, amplitude differs by sign, but so does $\bar{v}v$, so again get attractive potential.

• Example $\Gamma = i\gamma_5$, $N + \phi \to N + \phi$, simplify $i\mathcal{A}$. Compute $|\mathcal{A}|^2$ and average over initial spins and sum over final spins. Simplify.

$$i\mathcal{A} = ig^2\bar{u}^{(r')}(p')\not{q}u^{(r)}(p)F, \qquad F \equiv \left[\frac{1}{2p\cdot q + \mu^2 + i\epsilon} + \frac{1}{2p'\cdot q + \mu^2 + i\epsilon}\right].$$

$$|\mathcal{A}|^2 = g^4F^2q_\mu q_\nu Tr[\bar{u}(p')^{r'}\gamma^\mu u(p)^r\bar{u}(p)^r\gamma^\nu u(p)^{r'}].$$

Average over initial spins and sum over final ones

$$\frac{1}{2}\sum_{r,r'}|\mathcal{A}|^2 = \frac{1}{2}g^2F^2q_\mu q_\nu Tr[(\not{p}' + m)\gamma^\mu(\not{p} + m)\gamma^\nu]$$
$$= 2g^4F^2[2(p'\cdot q)(p\cdot q) - p\cdot p'\mu^2 + m^2\mu^2].$$